ASTR 688R
Problem Set 5
Due Thursday, May 4

Since you have only one week to do this problem set instead of two, there are two problems instead of four. This set is, however, worth as much as any other set: each problem is worth eight points.

1. Write a program to solve the Lane-Emden equation numerically. As before, I want a short write-up of the equations you used, hardcopy of the output (θ versus ξ for selected values of n [see below]), and an e-mail copy of your code, which as before has to be able to compile and run on the astro machines (I'm not going to install anything!).

(a) Check your results against the analytic solutions for $n = 0$, $n = 1$, and $n = 5$. Do this by plotting θ versus ξ for the analytical and for your numerical solution on the same graph (one graph per value of n) and by listing the value of ξ_1.

(b) What is ξ_1 for $n = 3$ and $n = 3/2$, the two most interesting values in practice?

(c) Solve the Lane-Emden equation for the Sun, assuming it is an ideal gas ($n = 3/2$). What is the central density, if the mass and radius are fixed at their actual values?

Hint: try the simplest numerical integrations first! I won't demand ultraprecise answers; three significant figure agreement with analytic solutions is fine.

2. (Binary stars). Suppose we have a system with a star of mass M_1 and a planet of mass $M_2 \ll M_1$ orbiting the star at a separation R. Find the location of the first three Lagrange points in the system, for test masses $m \ll M_2$ (this is a restricted three-body problem). These three Lagrange points are all in a line with M_1 and M_2, and are points where the total radial force (gravitational plus centrifugal) on the test mass is zero, where it is assumed that the test mass and both larger masses are orbiting in circles at an angular frequency $\Omega = \sqrt{G(M_1 + M_2)/R^3}$. Therefore, three masses in this configuration will continue to be in a straight line as the system rotates. L_1 is defined as the point between the two masses where the radial forces cancel; L_2 is defined as the point on the same side of M_1 as M_2 is, but farther away from M_1, where the forces cancel; and L_3 is the point on the opposite side of M_1 from M_2. Defining $\alpha \equiv M_2/M_1 \ll 1$, find the distance of each of the Lagrange points from the center of mass of the system, to lowest order in α. **Warning:** if for each point you define the radius to be $r = R(1 + \epsilon)$, with $\epsilon \ll 1$, then the locations of L_1 and L_2 can be determined without expanding quantities such as $(1 + \epsilon)^2$ past linear order in ϵ, but to find the location of L_3 to lowest order you need to go to quadratic order in ϵ.