ASTR 688R
Problem Set 3
Due Thursday, March 16

1. Suppose that a tenuous (optical depth much less than unity) fully ionized hydrogen gas is placed outside a spherical star of luminosity L and mass M at a distance r from the center of the star. Assume that the radiation from the star interacts with the electrons in the gas only via Thomson scattering, but that the electrons and protons are electrically coupled, so that they act as single particles with mass m_p and scattering cross section σ_T. Assume also that gravity is Newtonian, that the star is not rotating, and that the gas is initially at rest with respect to the star (i.e., the gas is not orbiting the star). Calculate the critical luminosity $L_{\rm crit}$ such that the radial radiation force exactly balances the inward gravitational force. What is the value of this luminosity for a star of one solar mass? Qualitatively, what would happen to this critical luminosity if the gas actually had a cosmic abundance of metals and the radiation opacity were higher than that given by Thomson scattering because of the contribution of absorption processes?

2. (**8 pts**) Leslie Sage, the astronomy editor for Nature, has asked you to review a paper that has just been submitted. The author is your good friend Dr. I. M. N. Sane, who has a new theory of extrasolar giant planets (of roughly the mass of Jupiter). Upon reading the paper you find that Dr. Sane has used a linear density model and has assumed that the composition of the central parts of giant planets is pure hydrogen. You also find that his model is fundamentally based on three premises about the centers of these planets: (1) that the free electrons in the centers of these planets are degenerate, (2) that almost all (>99%) of the pressure in the planet centers is due to degeneracy pressure, and (3) that conductive transport is much more important than radiative transport. In your report to Dr. Sage

about this paper, calculate the following for Jupiter (mass 2×10^{30} g, radius 7×10^9 cm):

(a) Using the linear density model from chapter 1, estimate the temperature and density at the center of Jupiter. From this and the Saha equation, estimate the ionization fraction and hence the number density of free electrons, assuming that hydrogen can be treated as a single-level atom with a binding energy of 13.6 eV. How accurate (qualitatively) is the Saha equation likely to be?

(b) Using the results of part (a), compute the degeneracy parameter E_F/kT for the free electrons.

(c) From the equations in Chapter 3 of Hansen & Kawaler, compute the degeneracy pressure and compare it to the pressure of an ideal gas in the center of Jupiter.

(d) Using the approximate expressions for the radiative opacity (remembering to take into account partial ionization!) and conductive opacity given in the book, compare the two and determine which channel dominates energy transport.

(e) If you assume (as Dr. Sane does) that the average density for any giant planet is the same as Jupiter's average density, what mass would the planet have to have in order for all of Dr. Sane's assumptions to hold?

3. Consider a gas of pure hydrogen in thermodynamic equilibrium at a density of $\rho = 0.1$ g cm^{-3}. Assume that hydrogen can be treated as an atom with just a ground state at a binding energy of 13.6 eV. How accurate is the Saha equation likely to be in this case? Use it and the various cross sections to determine approximate conditions for which of bound-free, free-free, or Thomson scattering will be most important at a temperature T and photon energy $h\nu > 20$ eV. What are the relative opacities for $T = 10^5$ K and $h\nu$=1 keV?