Lifetimes and timescales

Ask class: given that the duration of all phases of star formation greatly exceed the history of astronomy, how could we estimate relative durations? It is often done by relative numbers. **Ask class:** what could mess this up? Selection effects, such as the stars being more detectable during one phase than another. Some inferred time scales are: (1) pre-collapse molecular cloud, up to 10^7 yr, (2) collapse, $\sim 10^5$ yr, (3) post-collapse (PMS), $\sim 10^7$ yr (recall the Kelvin-Helmholz time). This collapse is the free-fall time, without support.

Observations of binaries

Most stars are in binaries (maybe 2/3 of the total), so from a numbers standpoint we should start with them! Binaries can tell us a lot more than single stars, about star formation and about other things. For single stars, we can get the stellar mass and some information about the circumstellar disk. For binaries, two masses (mass ratio, ages), orbit (separation, eccentricity), and disks (maybe two; possibly a circumbinary disk; coplanarity?).

Ask class: how do we determine that a pair of stars forms a binary? If it is a wide binary, it is a "visual" binary. The angular resolution is limited by either seeing or by diffraction. **Ask class:** suppose we have a binary at 100 pc, and we use a ground-based telescope. At about what separation could we resolve the two? With 1" seeing, 100 AU. With Hubble, 3 AU. Even better (factor of 4) with perfect adaptive optics on Keck.

Ask class: if the stars are too close to resolve, what is another way? Spectroscopic, by radial velocity variations. Complementary with visual binaries, because this only works for close separations (why? too far away means that the duration is long and the radial velocities are small). Mizar, in the Big Dipper, is a visual binary. When each member of the binary is examined more closely, it in turn is a spectroscopic binary.

Can also use speckle interferometry. Lots of short (<0.1 s) exposures, faster than atmosphere changes. Take average power spectrum, divide by that of a single star. Can get close to the image without distortion and use that way.

Finally, can use lunar occultation: two steps down in brightness when obscured, followed by two steps up on the way out. Can in principle get 0.005" to 1" that way. Limited to those regions in path of moon, which include Taurus and Ophiuchus regions.

Surveys of binaries have been hampered by selection effects. For example, it has often been said that most O stars in binaries have other O or B stars for companions. That may be, but detecting an M dwarf would be tough in those circumstances!

Nonetheless: it may be that the fraction in binaries increases with the mass of the primary (>42% for M dwarfs up to >70% for B and O stars). The distribution of periods for M, K, and G dwarfs goes from a few days to 10^8 days (10^4 AU), with a separation distribution like $dN/da \propto 1/a$, suggesting scale-free formation. The reason for a minimum separation is that the stars can't bump into each other (!). **Ask class:** what would be a reason for a maximum separation? If too far apart, would be disrupted by molecular clouds or passing stars.

Star Formation: Theory

Again, take a lot from Matthew Bate's lectures.

Average density of the galaxy is around 10^{-24} g cm^{-3}, average density of a star is ~ 1 g cm^{-3}, so obviously a rather substantial density increase has happened! On large scales, gravity dominates, so we have to think about gravitational collapse. For this to happen, gravity must, locally, dominate all support mechanisms. **Ask class:** what kinds of things might support gas against gravitational collapse? Thermal energy/pressure, rotation, magnetic fields, turbulence. Need to find a condition for when something collapses and when it doesn't.

We can, as theorists, imagine that we have set up a nonrotating, nonmagnetic, nonturbulent cloud and ask about when gravity will beat thermal pressure. This leads to a minimum mass called the *Jeans mass.* **Ask class:** for a uniform-density spherical cloud of mass M and radius R, what is the gravitational energy? $E_g = \frac{3}{5}\frac{GM^2}{R}$. **Ask class:** what is the thermal energy if there are N particles at temperature T? $E_t = \frac{3}{2}NkT$. The condition for gravity to win is $E_g > E_t$, so

$$\frac{3}{5}\frac{GM^2}{R} > \frac{3}{2}\frac{M}{\mu m_p}kT \ , \tag{1}$$

where $N = M/\mu m_p$. Therefore, need

$$M > \frac{5}{2}\frac{kT}{G\mu}R = \frac{5}{2}\frac{kT}{G\mu}\left(\frac{M}{\frac{4}{3}\pi\rho}\right)^{1/3} \Rightarrow M > \left(\frac{5}{2}\frac{kT}{G\mu m_p}\right)^{3/2}\left(\frac{4\pi}{3}\rho\right)^{-1/2} \equiv M_J \ . \tag{2}$$

This is $M_J \approx 2M_\odot T^{1.5}n^{-0.5}$. Therefore, if $M > M_J$ then the cloud will start to collapse, whereas if $M < M_J$ the cloud will not collapse. Note that this is the absolute minimum mass for a bound cloud. For $\rho = 10^{-23}$ g cm^{-3} and $T = 100$ K, typical of the ISM, neutral hydrogen ($\mu = 1$) has $M_J \approx 10^4\, M_\odot$.

Suppose first that there is no support against collapse. In that case it would collapse on a free-fall timescale

$$t_{ff} = \left(\frac{3\pi}{32G\rho}\right)^{1/2} = 3.4\times 10^7 n^{-1/2}\,\mathrm{yr} \ . \tag{3}$$

Typical densities in sterile (non-star-forming) regions are $\sim 50 - 100\ \mathrm{cm}^{-3}$, implying $t_{ff} \approx 5 \times 10^6$ yr. This is $\sim 0.1\times$ the inferred lifetime of clouds, so something must hold up the collapse.

As the gas cloud collapses, its thermal energy goes up and can in principle halt (or at least slow) the collapse. We have

$$M_J \sim T^{3/2}\rho^{-1/2}\ . \tag{4}$$

If the equation of state is polytropic, so that $P \propto \rho^\gamma$ and $T \propto \rho^{\gamma-1}$ then

$$M_J \sim \rho^{(3\gamma-4)/2}\ . \tag{5}$$

Ask class: what does this mean for whether collapse will stop or run away? If the Jeans mass decreases as the cloud collapses, there will be a runaway; if it increases, the collapse will stop. Therefore, for $\gamma < 4/3$ there is a runaway.

Halting collapse by rotation

Here, it's a comparison of the rotational energy with the gravitational energy:

$$\beta = \frac{E_{\rm rot}}{|E_{\rm grav}|} \ll 1 \tag{6}$$

is the usual initial condition. Now, $E_{\rm rot} \sim (R\Omega)^2$ and $E_{\rm grav} \sim 1/R$, so $\beta \propto R^3\Omega^2$. If angular momentum is conserved during the collapse, then L=const=$R^2\Omega$, so $\Omega \sim 1/R^2$ and $\beta \propto 1/R$. Since the collapse is over many orders of magnitude (say, 10^{18} cm for a solar mass cloud to 10^{11} cm for a star), this means that rotation can halt the collapse even if it is unimportant initially.

Let's work this out. The Galaxy rotates with a period of about 200 million years, so let's say that the initial molecular cloud shares at least that rotation. If a 1 $M_\odot$ portion of the gas has a radius of 10^{18} cm initially, then to collapse to something the size of the Sun (10^{11} cm) and conserve its angular momentum it needs to spin 10^{14} times faster, or in about 1 minute(!) compared with ~3 hr for breakup and ~30 days for the actual rotation period of the Sun. This is a serious problem! As a sidelight, let's think for a second about what other angular momentum exists in the solar system. **Ask class:** do they know what fraction of the mass of the solar system is in the Sun? About 99.8%. What fraction of the angular momentum? The Sun rotates at about 1/300 of the Keplerian orbital frequency at its radius. Jupiter orbits at Keplerian, of course. In addition, Jupiter is at a radius of 5 AU=7×10^13 cm, or 1000 times the radius of the Sun, so it has a specific angular momentum about $1000^{1/2}$ times greater than a particle orbiting at the limb of the Sun. Therefore, the specific angular momentum of Jupiter is about $1000^{1/2} \times 300 = 10^4$ times that of the Sun. The Sun's mass is 10^3 times Jupiter's so Jupiter's angular momentum is 10 times that of the Sun. In reality, the Sun's mass is centrally concentrated, and $J_J/J_\odot \approx 100$. So, that helps, but not enough.

Magnetic support against collapse

Again, compare the magnetic energy to the gravitational energy. **Ask class:** what is the magnetic energy density for a field of strength B? $E_B = B^2/8\pi$. Then the condition is $\frac{3}{5}\frac{GM^2}{R} > \frac{B^2}{8\pi}\frac{4}{3}\pi R^3$, so

$$M^2 > \frac{B^2}{3.6G}R^4 \Rightarrow M > \frac{BR^2}{(3.6G)^{1/2}} \equiv M_J(\mathrm{mag}) \; . \tag{7}$$

If the gas is perfectly conducting, so that there is flux freezing and BR^2=constant, then magnetic field cannot halt collapse once it starts. Typical values are:

(1) ISM, $n \sim 10$ cm^{-3}, $B \sim 10^{-6}$ G, $T \sim 100$ K

$$M_J(\mathrm{therm}) \sim 10^4\, M_\odot \sim M_J(\mathrm{mag}) \; . \tag{8}$$

(2) Giant molecular clouds, $n \sim 10^2 - 10^3$ cm^{-3}, $B \sim 3 \times 10^{-5}$ G, $T \sim 10$ K

$$M_J(\mathrm{therm}) \sim 10\, M_\odot \, ; M_J(\mathrm{mag}) \sim 10^3\, M_\odot \; . \tag{9}$$

(3) Dense cores, $n > 10^4$ cm^{-3}, $B \sim 10^{-5}$ G(?), $T \sim 10$ K

$$M_J(\mathrm{therm}) \sim \mathrm{few}\, M_\odot \sim M_J(\mathrm{mag}) \; . \tag{10}$$

At those densities, though, magnetic field may be decoupled.

We have therefore identified three potential problems with star formation by gravitational collapse: (1) if the polytropic index exceeds 4/3, the cloud can heat up fast enough to stop collapse with thermal pressure, (2) rotation and a centrifugal barrier will generally set in if the cloud conserves its angular momentum, (3) in some cases the magnetic field may halt collapse. We will now consider ways out of these problems.

Heating and Cooling

First, the thermal problem. A molecular cloud is heated by external radiation (X-rays, gamma-rays, UV) when it is low-density, and by cosmic rays more generally. At low densities and high temperatures, cooling is relatively inefficient. It tends to proceed via molecular radiation, such as from H_2 and CO. At low temperatures and high densities, cooling from dust grains dominates. This radiation occurs in the IR because that's where it is able to escape. This is why IR mapping tends to track dust.

The net result is that when a cloud becomes optically thick (say, $\tau > 1/2$), then it is self-shielded from external radiation and the interior portions of the gas can cool in peace. This happens by formation of molecules, for example, and the equilibrium temperature drops to about 10 K. Therefore, the inner part of the gas can radiate away the energy it gets from gravitational collapse ("settling" might be more appropriate), and continue to contract.

Angular momentum

As we indicated before, the specific angular momentum ("specific" means "per mass", so it's L/M) of giant molecular clouds is vastly greater than that of stars, so you have to get rid of most of it. For a $\sim$1 pc GMC, $L/M > 10^{23}$. For a dense cloud core, $\sim$0.1 pc, $L/M \sim 10^{21}$. For the Sun, $L/M \sim 10^{15}$. Lots of orders of magnitude. Where does the excess go?

You might think it could go to binaries, or planets, or that young stars might have a lot of angular momentum, but it isn't so. A 3-day binary has $L/M \sim 10^{19}$. **Ask class:** how would the angular momentum go with orbital period? Like $P^{1/3}$, so even at a 10^4 yr binary, $L/M \sim 10^{21}$. We already found that Jupiter doesn't have enough either, $\sim 10^{20}$. Young stars like T-Tauris have $L/M \sim 10^{17}$, so more than the Sun but nothing close to that of the initial GMC.

Therefore, specific angular momentum must be transported away from the system entirely. **Ask class:** what are some ways that this can happen? Winds (magnetic, especially), jets, disks. Still a lot of discussion about how this happens. In somewhat more detail, angular momentum can be removed by:

(1) Magnetic braking. If a magnetic field threads a cloud, it will try to enforce uniform rotation. This moves angular momentum outward. This can happen before the collapse of the cloud. It is also effective in slowing down the rotation of stars.

(2) Collapse to a disk. Stresses within the disk transport angular momentum outward and mass inward.

(3) Star-disk coupling. If the star has a significant magnetic field, it can get slowed down by interaction with the disk (or spun up, for that matter). Along with the class, compute the strength of the field necessary for a particular radius of coupling. For example, how strong a field would be necessary so that the orbital frequency at the coupling radius was that of the Sun?

Magnetic fields: ambipolar diffusion and reconnection

Remember that magnetic fields are only an important support mechanism if the mass is subcritical relative to the magnetic Jeans mass. Masses in excess of this can collapse without significant hindrance from the field.

The gas contains both charged and neutral matter. The ions can be tied to the field, but the neutrals move freely through it. The two components interact by mutual collision. By balancing the gravitational force on the neutral component with the drag produced by the ions (assumed tied to the field), people have estimated a time for ambipolar diffusion:

$$t_{\rm AD} \approx 5 \times 10^5 (X/10^{-8})\,{\rm yr}\ , \qquad (11)$$

where X is the ionized fraction. This probably plays a role in many clouds, but there is dispute about whether this is the final word: the time scale for formation may be too large, and since the magnetic Jeans mass is $\sim 10^3\,M_\odot$, lower-mass cores could be tough to form.

Another possibility is reconnection. As the magnetic field is compressed by the matter (which wants to collect in the gravitational well), the field can change its topology by forming small closed loops. If this happens, the field does not provide as much support against large-scale motions of the matter. In addition, field may self-annihilate and reduce ratio of magnetic flux to mass. But, not well-studied.

Shu and collaborators have suggested that subcritical masses may form low-mass stars, and supercritical may collapse to form high-mass stars.