Specific fusion reactions

Start off: could have homework due on Tuesday instead of Thursday. Ask how the homework went (homework should be turned in at the beginning of class). Ask if anyone attempted the extra credit problem. Show them how to do problem 4 (global cooling) without ever introducing the Earth's orbital radius. Of course, will give full credit for solutions that *do* perform the whole calculation! Should also give the transparency part of the Gamow peak discussion. Work in explosive burning as in Type Ia supernovae, ask if there are questions. **Mention:** double counting with identical particles.

Proton-proton reaction. **Ask class:** for most stars, burning of what element is likely to provide most of the energy? (Hydrogen, of course, because of abundance and also because of the high binding energy liberated). For most stars (stars on main sequence), the burning of hydrogen has the net effect

$$4^1H \rightarrow He\ . \tag{1}$$

Ask class: Is this reaction, as written, likely to occur in stars? No, because this would require an extremely improbable reaction: four protons hitting each other all at once, and when doing so simultaneously undergoing inverse beta decay! Won't happen. So, need to take this in smaller steps.

Consider first the case in which only protons are around. Then see if the class can figure out a way in which two protons can react to release energy (the hint is that they need to form a nucleus, but two protons can't do that, so one has to have an inverse beta decay to a neutron, which can only happen when they are close enough to overlap!). Therefore, the proton-proton reaction is

$$^1H +^1 H \rightarrow^2 H + e^+ + \nu_e\ . \tag{2}$$

Even this is extremely improbable.

The rate for this reaction is

$$r_{p-p} = \frac{1.15\times 10^9}{T_9^{2/3}} X^2\rho^2 e^{-3.38/T_9^{1/3}}\ \mathrm{cm}^{-3}\,\mathrm{s}^{-1}\ . \tag{3}$$

Define: X is the mass fraction of hydrogen, Y is the mass fraction of helium, and Z is the mass fraction of everything else combined (obviously $X + Y + Z = 1$; typically $X \approx 0.7$, $Y \approx 0.3$, and $Z \approx 0.03$). In these sorts of expressions, things are expressed in cgs units: therefore, the density ρ in this case is in units of g cm^{-3}. **Ask class:** How would we estimate the rate of energy generation in this case? Reasonable estimate is the binding energy of deuterium, which is a *total* (not per nucleon) of 2.22 MeV compared with the summed mass of a neutron and a proton, but only about 1.5 MeV compared with the summed mass of two protons. However, it's a little tricky because of the energy in neutrinos, which departs forthwith (10^{-44} cm^2) and contributes nothing to energy generation, although plenty of information is available that way. **Maximum energy generation rate at 100 g/cc compared with burning a sheet of paper? Maximum is less than $10^9\rho$ cm^{-3} s^{-1}, which at 2 MeV=3×10^{-6} erg a pop is *tiny* compared to the $\sim 4\times10^{11}$ erg g^{-1} of combustion! Ask how this is consistent.** At this point, burn normal paper. Then do flash paper, relate to fast deflagration.

The temperature exponent is

$$\nu_{p-p} = \frac{11.3}{T_6^{1/3}} - \frac{2}{3}\ , \tag{4}$$

which is much less sensitive than some of the reactions we saw last time (at the solar center, where $T_6 \approx 15$, $\nu \approx 4$). The mean life of a proton against destruction by this mechanism in the Sun is 6×10^9 yr, which (not coincidentally) is close to the main sequence life of the Sun. **Lesson from this:** when weak decays are required (as in this case, p$\longrightarrow$n), the reactions are slow.

Might think that the next step is to combine two deuterium nuclei to make ^{4}He, which would be perfectly legal. But the number of protons is huge, so once deuterium is formed it almost immediately captures a proton and becomes more bound as ^{3}He. The p-p chains are given on page 236 of HK (as temperature increases, the chains become more important as I, II, III). Overall energy release by these three chains depends on relative weighting, but to leading order is

$$\epsilon_{\rm eff} \approx \frac{2.4\times10^4\rho X^2}{T_9^{2/3}} e^{-3.38/T_9^{1/3}}\ {\rm erg\,g^{-1}s^{-1}}\ . \tag{5}$$

Note that the $^8Be \rightarrow ^4 He + ^4 He$ reaction has a 10^{-16} s lifetime!

The equilibrium abundance of D from all this is incredibly tiny, less than $D/H = 10^{-17}$ for temperatures in the stellar range. But the amount of deuterium in the universe is way more than this: several times 10^{-5} in some interstellar clouds and in the direction of quasars, and a few times 10^{-4} in the oceans. Something's wrong! Need another way to produce deuterium (**Ask class**). A way you might think of is spallation, but it turns out that BBN is the only viable way people have found.

Big bang nucleosynthesis: In the early universe, at high temperatures ($z\sim10^{10}$, $T\sim3\times10^{10}$ K), free neutrons can exist because there is a soup of protons, electrons, and neutrinos with enough energy. Can get $n+p\rightarrow d+\gamma$, but the reverse reaction destroys deuterium just as fast. At cooler temperatures, 10^9 K, the deuterium abundance can grow, then it captures more protons and ^{4}He is produced. Not enough time to produce too much that is heavier, because of the lack of any stable nucleus of mass 5 or mass 8. There is also lithium produced, and a couple of other light elements. BBN is considered one of the great successes and tests of the hot big bang model, but measurement of primordial abundances is extremely difficult because many things (like deuterium) can be destroyed and others (like ^{3}He) can be both created and destroyed.

Tell story of CMB: Gamow initially thought all elements could be produced this way. Abundance of helium suggested a rough value for the CMB temperature. In 1965, a group at Princeton led by Robert Dicke was gearing up to detect such a thing (actually looking for evidence of element *destruction* in an oscillating universe!), and a colloquium was given at Johns Hopkins about it because they figured no one else could catch up. Little did they know, just 30 miles from Princeton, in Holmdel, NJ, Penzias and Wilson were puzzling over excess noise in their antenna ("white dielectric material", birds nesting), which turned out to be just the CMB! They got the Nobel for it.

The p-p chains aren't the only way to burn protons to helium. In fact, at higher temperature the CNO cycles are more important, because they have a stronger temperature dependence (recall general discussion of last time). Page 239, Hansen & Kawaler. The first one is a cycle in the sense that you can start virtually anywhere in it and come back eventually. **Ask class:** what would a way be that the cycle could be broken? (for example, alpha capture onto ^{16}O). Can also supply it by making carbon, which we'll learn later. Key reaction in both cycles is $^{14}N(p,\gamma)^{15}O$. Slow, and appears in both cycles.

The importance of this process is partially responsible for the abundance of these elements. In fact, the mass fraction of the three combined is about 3/4 of the total mass fraction of "metals" (Z¿2). The energy generation rate for the CN or CNO cycles is

$$\epsilon_{\rm CNO} \approx \frac{4.4\times 10^{25}\rho X Z}{T_9^{2/3}} e^{-15.2/T_9^{1/3}}\ {\rm erg\,g^{-1}\,s^{-1}}\ . \tag{6}$$

Point out: X and Z dependence here, versus X^2 dependence for p-p. Energy generation rate is much higher than p-p, and the temperature exponent is

$$\nu({\rm CNO}) = \frac{50.8}{T_6^{1/3}} - \frac{2}{3}\ , \tag{7}$$

or about 18 for $T = 2\times 10^7$ K. Diagram of CNO versus pp energy generation rate. **Ask class: what is the limit of energy generation this way, i.e., how long could it operate?** Answer: can't get more energy than is available!

Triple-α reaction: Helium burning. Problem is that there is no stable element of mass 8, so the first step ($\alpha+\alpha\rightarrow^8 Be$) is endothermic by 92 keV. Therefore, for this reaction to occur at all, the Gamow peak has to be around this. For $Z_\alpha = 2$, $\mu = 2$, $E_0 = 3.9T_6^{2/3}$ keV, implying that this gets to $\sim$90 keV when $T\sim 10^8$ K, so that's the kind of temperature you need. Can then calculate the equilibrium concentration of ^{8}Be and then go on to the next step in the reaction, which is the capture of another α by the ^{8}Be nucleus. The excited state of carbon that is formed usually decays right back into α+^{8}Be, but sometimes it gets to the ground state, which then forms carbon.

Ask class: think of differences in how hydrogen can be burned, at least at first, for the first generation of stars (essentially no metals) versus stars formed now. A big difference is that there won't be CNO early on, so pp-cycle has to do its job until C can be formed via the triple-α process. The energy generation rate is

$$\epsilon_{3\alpha} = \frac{5.1 \times 10^8 \rho^2 Y^3}{T_9^3} e^{-4.4/T_9} \, \mathrm{erg\, g^{-1}\, s^{-1}} \; . \tag{8}$$

The density and temperature exponents are

$$\lambda_{3\alpha} = 2, \qquad \nu_{3\alpha} = \frac{4.4}{T_9} - 3 \; . \tag{9}$$

Very temperature sensitive ($\nu \approx 40$ when $T = 10^8$ K).

Carbon, Neon, and Oxygen: Here the reactions tend to get more diverse. For example, carbon can capture an alpha and become oxygen: $^{12}C(\alpha,\gamma)^{16}O$, and oxygen can become neon in the same way: $^{16}O(\alpha,\gamma)^{20}Ne$. Can also have nuclear reactions between two heavy nuclei, such as $^{12}C +^{12} C \rightarrow^{20} Ne + \alpha$ or $\rightarrow^{23} Na + p$, or $^{16}O +^{16} O \rightarrow^{28} Si + \alpha$. **Ask class:** would these reactions be expected to happen at higher or lower temperatures than what we discussed previously, and would they be expected to be more or less temperature sensitive? Higher temperature and more sensitive because of the large nuclear charge. For example, the exponential part of the $^{12}C +^{12} C$ reaction (summed over channels) is like $e^{-84T_9^{-1/3}}$, and the leading factor for $^{16}O +^{16} O$ is $e^{-136T_9^{-1/3}}$!

Photodisintegration: At such high temperatures (around 10^9 K), photons in the tail of the Planck distribution are energetic enough to photodissociate some nuclei. For example, can have $^{20}Ne(\gamma,\alpha)^{16}O$ as well as the reverse process. Can also have several reactions in succession, like $^{20}Ne(\alpha,\gamma)^{24}Mg(\alpha,\gamma)^{28}Si$. In general, things therefore get more complicated.

Heavy elements: Above 3×10^9 K, things get *really* involved. For example, $^{28}Si(\gamma,\alpha)^{24}Mg(\alpha,p)^{27}Al(\alpha,p)^{30}Si$, which effectively has added two neutrons to the original silicon nucleus. This kind of stuff, taken upwards to the "iron peak" elements which have the highest binding energy per nucleus, constitutes silicon burning but is awfully complex. The equilibrium nucleus is ^{56}Fe, but in some cases (e.g., supernovae) the time scale is so short that instead ^{56}Ni is produced (because it has an equal number of protons and neutrons and can therefore be built up from things like ^{28}Si without waiting for weak interactions or electron captures, which are slow).

Another thing that happens at these extremely high temperatures, as we saw earlier with BBN, is that neutrinos (electron family) can be generated (why? Mainly because e^-e^+ pairs are energetically possible, and if these annihilate then neutrinos can be produced). They escape immediately, giving the star no net energy and speeding its doom :). **Mention: neutrino production in gamma-ray bursts**.

Finally, if a large iron core is built up, no net energy can be generated in the core, and a supernova results. But that's for later.

Summary: Proton burning, which supplies most of the energy for most of the time because of the binding energy curve, can happen in two ways: the pp chain and the CNO cycles. The latter is more important for higher temperatures. Helium burning happens via the triple-α process. Burning of heavier elements can happen by alpha capture or by a number of other processes, which multiply as the temperature increases. Neutrino production at high temperatures takes energy away from the system in a very efficient manner.