Stellar energy sources

Of course nuclear fusion will be it, but in this lecture I want the students to come up with that realization and know why other things don't work. I also want them to know that all of the candidates on the list except for burning could be the dominant reservoirs of energy in some astrophysical sources (Fusion:stars. Fission: Earth. cooling: some white dwarfs. Crystallization: some white dwarfs, maybe Earth. Magnetic fields: magnetars. Rotation: isolated pulsars. Gravitational energy: any accreting source.)

First get them to come up with a list of candidates. Stress to them that in this kind of process, you first jot down possibilities without regard to whether they are realistic, then examine them critically. Possibilities: fusion, fission, cooling, crystallization, burning, magnetic fields, rotation, gravitational energy.

Next, ask them what we need to explain. The point is that the Sun has a luminosity of about 4×10^{33} erg s^{-1}, and from fossil records we know that it has been shining at this rate for something like 5 billion years, so that's around 5×10^{50} erg. Whatever energy source there is must be able to provide at least this amount.

For each candidate mechanism, ask the class what information is necessary to evaluate it. Then, do calculation.

Burning

Quantity needed is energy released per gram of burning. This is about 10^4 cal g^{-1}, and since 1 cal$\approx$4 J$=4 \times 10^7$ ergs, that's 4×10^{11} erg g^{-1}. The total mass of the Sun is 2×10^{33} g, so the total energy available is 8×10^{44} ergs, which is much too small. Can get this approximate value from first principles by figuring that burning is a chemical reaction, so it will release something like 1–10 eV per reaction, or maybe about 1 eV per nucleon. That gets the rough order of magnitude of the energy release.

Crystallization

Again, what is energy per gram? In this case one can again see that things are more or less electronic, in that it's electrostatic interactions that determine the energy. Since burning failed by such a large margin, crystallization will as well. If slightly more accuracy is needed, note that crystallization=freezing (more or less), and as a result the ions have fewer degrees of freedom than they did before. Typically you have something like $\frac{1}{2}kT$ per degree of freedom, so at some melting temperature T_m you'll release around $\sim kT_m$ per ion. A typical freezing temperature for metals is a few thousand degrees, so 0.1–1 eV per ion is released, even less than for burning.

Cooling

From the virial theorem, the total internal energy of the Sun is of the order of GM^2/R, or about 4×10^{48} ergs. This falls short by a factor of 100.

Rotation

How fast could the Sun possibly rotate? At the break-up speed, at which each element in the Sun is moving at a Keplerian velocity. But, by the virial theorem, the total kinetic energy is again comparable to GM^2/R, which is way short of what is needed. In addition, the visible layers of the Sun rotate at frequencies 100 times less than the Keplerian frequency, and helioseismology results indicate that the central part of the Sun is also rotating at dramatically sub-Keplerian rates.

Magnetic fields

What is the average magnetic field in the Sun required to have the necessary energy? Energy density in B fields is $B^2/8\pi$, so we need $E = (B^2/8\pi)\frac{4\pi}{3}R^3 = \frac{1}{6}B^2(7\times10^{10})^3 = 5\times10^{50}$. That means that the average field must be about 3×10^9 G. That is much larger than the field in even sunspots, which can get to $\sim$1000 G, but this by itself doesn't disprove things, because maybe the internal field is much greater than the external (although there are problems with this as well...). To be a little more convincing, we need another argument. If the pressure were dominated by the magnetic field, the star would be unstable. So, we need the matter pressure to be at least as great as the magnetic pressure. The magnetic pressure is $P_B = B^2/8\pi = 5\times10^{17}$, and if we approximate the matter by an ideal gas then it is $P_m = nkT$, where $T\sim10^7$ K. Equating the two means that the average number density is $n = 5\times10^{26}$ cm^{-3}, implying a density of $\rho = 700$ g cm^{-3}. That's 500 times the density of the Sun, so no way.

Gravitation

$E_G = -GM^2/R = 4\times10^{48}$ ergs. Short by a factor of 100, as was cooling. Appear to be satisfied by this, but then come back and be puzzled by the possibility that some part of the mass of the Sun is actually much denser, like a white dwarf or neutron star. If half of the Sun were a neutron star, energy release from accretion would be 10^{20} erg g^{-1}. That implies an accretion rate of 4×10^{13} g s^{-1}, or about 10^{31} g (=0.5% of the Sun's mass) in its lifetime. Can't rule this one out so easily. Bring in here the story of Landau and Oppenheimer (resolution is not so easy to see: such a star would look like a supergiant), and Thorne-Zÿtkow objects. Also bring up Shu's point that, due to the endothermic production of heavy elements in a supernova, the net result is that the energy has been primarily gravitational!

Matter-antimatter reactions

There is plenty of energy available, in principle at least. To produce the $\sim10^{51}$ erg necessary, 10^{30} g of matter+antimatter needs to be annihilated.

That's 5×10^{-4} of the Sun's mass, so it's a small fraction. However, if this were spread diffusely throughout the star then it would all react very quickly and would blow the star apart. If it were concentrated in a central core, then the luminosity would be limited to the Eddington luminosity of about $10^{38}(M/M_\odot)$ erg s^{-1} at which radiation forces would prevent additional matter from falling in. However, as with gravitation onto a dense central core, the Sun would end up like a supergiant. Therefore, this too is not trivial to rule out, but nonetheless it can't power the Sun.

Hamsters

Suppose we were to power the Sun with hamsters on treadmills, or with their body heat. The average hamster apparently puts out about 1/3 Watt, or 3×10^6 erg s^{-1}. Suppose for argument's sake that a hamster has a mass of 10^3 g. Then in order to power the Sun's 4×10^{33} erg s^{-1} you'd need about 10^{27} hamsters, or about 10^{30} g total. That means that, mass for mass, hamsters are about 2000 times more efficient in generating energy than the nuclear fusion in the Sun! This is because, as we'll see later, fusion in the Sun via the pp chain proceeds very slowly. However, it can continue over $\sim 10^{10}$ yr, unlike the average hamster. Also unlike the average hamster, the Sun doesn't need to be fed.

Now suppose that we consider hamsters of different sizes. A typical mammal spends most of its energy keeping a constant body temperature, and hence the energy output scales like its surface area, or $M^{2/3}$ if we make the standard approximation of a spherical (or at least non-fractal) hamster. Thus, the energy output per mass goes like $M^{-1/3}$, so the larger the hamster the less efficient its output per mass. If we wanted to power the Sun with a single uberhamster, we would need a single hamster about $(10^{27})^{3/2}$ times more massive than a standard Terran hamster, or about 10^{43} g give or take a factor of a few. That's about the mass of the Large Magellanic Cloud, and at typical hamster densities would stretch out from the Sun to about the orbit of Saturn. Going in the other direction, suppose we enlisted femtohamsters of mass about 10^{-15} of the typical hamster. The total mass of femtohamsters required to power the Sun would be about 10^{25} g, or about the mass of Pluto. Unfortunately, however, none of the hamster family would be able to go on very long, and there'd be an astronomical mess to clean up.

Fission

Fission releases approximately 1 MeV per nucleon, or about 10^{-3} of the rest mass energy. For the Sun the rest mass energy is about 2×10^{33} g c^2, or 2×10^{54} erg, so 0.1% of that is 2×10^{51} erg, which is enough. So why couldn't fission be a viable mechanism? Two reasons. First, it would require that a large fraction of the Sun, at least 25%, would be made up of heavy elements such as uranium(!). Not so. Even more conclusively, fission is a process that has a critical mass, and you'd better believe that a solar mass exceeds that critical mass! Therefore, if you did set up that much uranium, it would blow itself to bits within a ridiculously short time. So no dice on this mechanism.

Fusion

Fusion of hydrogen to helium releases about 7 MeV per nucleon. That's plenty of energy. The main fuel, hydrogen, is the most abundant element in the universe, so that isn't a problem. The mechanism is self-regulating, because if too much energy is released there is a slight expansion of the star, cooling it and decreasing the energy generation rate (and vice versa). The only question is how this actually works, because there are many important details about how hydrogen burns to helium that are essential for being really convinced that this works. This, and discussions of other types of nuclear burning, will be the subject of the next few classes.

Thermonuclear fusion

Energetics. Define binding energy. Binding energy curve (H=0, He=7.1, C=7.7, Fe=8.8 MeV, all per nucleon. Drops to 7.5 near Uranium; see pg. 215). Approximate density (about 1.5×10^{14} g cm^{-3}). Coulomb barrier: distance is about 1 fm, so $e^2/10^{-13}$ cm=1.4 MeV. Maxwell-Boltzmann curve, exponential part is $e^{-E/kT}$, which is about e^{-1000}! Need tunneling.

Another contribution, which does not enter here directly but has importance in many other contexts, is the Fermi energy. As we will see later (and as they may have seen before), the Fermi energy is (among other things) the energy below which there is almost complete occupation by fermions. This can be related to a Fermi momentum using the uncertainty principle:

$$\Delta p \Delta x \sim \hbar \ . \tag{1}$$

This means, more or less, that the momentum cannot be less than about $\hbar/\Delta x$. Thus, if some species of particle has a number density n, so that $\Delta x \sim n^{-1/3}$, then the momentum isn't less than $p_F \sim \hbar n^{1/3}$. If this is nonrelativistic, then $E_F \approx p_F^2/2m$, and if relativistic then $E_F \approx p_F c$. This is real energy, in the sense that the energetic "cost" having a particle includes the Fermi energy.

We can now qualitatively understand two things: the general structure of the binding energy curve, plus some things about neutron stars.

Try to lead class through this: The binding of nuclei is a competition between two forces: the strong force, which binds nuclei together, and the electrostatic force, which repels them. An essential point is that the strong force has a very short range (of order a Fermi, 10^{-13} cm), whereas the electrostatic force has an infinite range (by which I mean that it doesn't die off exponentially). Starting off with small nuclei, adding nucleons increases the binding energy, making the nucleus more stable. When the nucleus gets large enough, however, the electrostatic repulsion works through the whole volume, whereas the strong force is only local, so electrostatic repulsion starts to win and the binding energy per nucleon decreases (it does not become positive, but it decreases). This can lead to instability and radiation such as alpha-particle, or fission. Could also mention nuclear magic numbers (don't ask them to derive); recent synthesis of ^{48}Ni, which is doubly magic but ridiculously proton-rich.

In neutron stars, as density gets higher the Fermi energy of electrons increases more rapidly than of p or n (because of m in denominator). Becomes relativistic at about 10^6 g cm^{-3}. It becomes energetically less favorable to have both p and e$^-$ (two particles) versus n, so equilibrium nuclei at high densities are extremely neutron-rich: e.g., 40 protons, 80 neutrons. At a density of about 4×10^{11} g cm^{-3}, free neutrons are stable and you have "neutron drip" in which free neutrons coexist with nuclei. In core, have few$\times 10^{14}$ g cm^{-3}, and could have $\sim$90% neutrons plus charge-balance of p and e, or (for same Fermi energy reasons) could have other exotic particles or states of matter.

Later, perhaps during the lecture on heavy element burning, I need to point out (Clayton pg 519) that when you get to really heavy elements, the Coulomb barrier is large enough that you have lots of photodisintegration and hence rearrangement of the elements. There is also substantial neutrino emission above 3e9 K, so the net rate of energy generation is small.