Neutrinos

Hansen & Kawaler, section 8.3.

Neutrinos have the property that they barely interact with the Sun on the way out, so when they are detected it's a direct measure of what's happening in the interior. **Ask class:** generically, on the nucleon level, what can produce neutrinos? Nuclear burning, but more precisely any conversion of a proton to a neutron or the reverse. **Ask class:** Why? Because of conservation laws: proton to neutron needs an electron somewhere to conserve charge, and the electron must be balanced by a neutrino to conserve lepton number.

Examples:

1. ^{1}H+^{1}H$\longrightarrow^2$H+e$^+$+ν_e. The pp reaction, emitting neutrinos in a continuum up to 0.42 MeV. From the PP-I chain.

2. ^{8}B$\longrightarrow^8$Be+e$^+$+ν_e. Also gives a continuum of energies, up to 15 MeV.

3. ^{7}Be+e$^-$ $\longrightarrow^7$Li+$\nu_e + \gamma$. Line neutrino, depending on Li nucleus state: 0.862 MeV (90%) or 0.384 MeV (10%).

4. CNO reactions: ^{13}N$\longrightarrow^{13}$C+e$^+$+ν_e (continuum, up to 1.3 MeV); ^{15}O$\longrightarrow^{15}$N+e$^+$+ν_e (continuum, up to 1.73 MeV); ^{17}F$\longrightarrow^{17}$O+e$^+$+ν_e (continuum, up to 1.74 MeV).

There are also rarer reactions that produce neutrinos: ^{1}H(pe$^-$,ν_e)^{2}H ("pep") produces line neutrinos at 1.44 MeV, and the "hep reaction ^{3}He(pe$^-$,ν_e)^{4}He produces a continuum up to 18.8 MeV. More than thirty years ago, Ray Davis and collaborators decided to detect these. Their approach was to use cleaning fluid containing chlorine (600 tons of it!) and every once in a while clean it out to see if they detect argon, which should be produced by:

$$\nu_e +^{37} \mathrm{Cl} \longrightarrow e^- +^{37} \mathrm{Ar}\ . \tag{1}$$

The neutrino must have an energy of at least 0.81 MeV. This means that the first of our examples (continuum neutrinos from the pp reaction) are undetectable by this method, although they are the most common neutrinos produced (>90% in the standard model). The rates are expected to be mighty tiny: if a "target" is defined as an individual chlorine atom (in this example), then a typical rate of detection is 10^{-36} captures s^{-1} target^{-1}, which is called a "solar neutrino unit" or SNU.

Well, what's the net result? The result is a puzzle that has persisted for a good thirty years: fewer neutrinos are detected than were expected. For example, the Homestake Mine results (from Davis et al.) get a flux of about 2 ± 0.3 SNU (1σ), compared to the predicted 7.9 ± 2.4 SNU (3σ). Subsequent experiments confirm the deficit. For example, SAGE and GALLEX are ^{71}Ga(ν_e,e$^-$)^{71}Ge experiments with 0.23 MeV thresholds, and can therefore detect at least some of the pp neutrinos. Depending on the energy and the reaction, a typical deficit is a factor 2-3 compared with expectations.

As this deficit has become more and more established, two types of solutions have been proposed. One is the class of astrophysical solutions, by which the temperature, density, or composition structure of the Sun is assumed slightly different than the standard solar model. Since some of these reactions (especially the CNO reactions!) are very temperature-sensitive, this seems promising (indeed, getting the right answer to a factor of 2 or 3 is impressive in its own right!). However, specific solutions have run into great difficulty, because of the huge number of observational constraints (most recently from helioseismology). The other type of solution is through particle physics, by assuming that (since most experiments see only electron neutrinos) there is an oscillation between flavors. If so, this would be a deviation from the standard model of electroweak interactions, and would hence be profoundly interesting to particle physicists!

For more details, see Dar & Shaviv 1999, Phys. Rept. 311, 115 (astro-ph/9808098), Haxton 1999, Nucl. Phys. Proc. Suppl., 77, 73-80, or any of a number of Bahcall articles.

A last point about neutrinos is that they are of course generated in enormous quantities in supernovae, where most of the protons and electrons squish together to form neutrons. Neutrinos (about 20 total) were seen from SN 1987A; if a similarly close one were seen today, about 100x as many would be observed because of the greater range of detectors (especially Super-Kamiokande). These can in principle tell us a huge amount about supernovae.

Helioseismology and asteroseismology

Hansen & Kawaler, chapter 10. Also, Brown & Gilliland 1994, ARA&A, 32, 37.

Hansen & Kawaler go into this subject in extreme depth. Here we'll cover only the basics (I will not expect you to be able to calculate at the HK level). After a quick overview, we'll sketch some of the perturbation techniques used for the mathematical analyses.

Asteroseismology (the general term for the study of normal-model pulsations in stars) contains a lot of information, but only recently have the data been of sufficient quality to inform us about the interior of the Sun and a few other stars. Some of the bits of physical information include the sound speed as a function of depth in the Sun, the variation of angular velocity with both depth and latitude (it's close to rotation on cylinders, but not quite), the depth of the convective envelope, and the helium abundance in the convective zone. It has also been used to provide evidence for differential rotation in white dwarfs and the presence of magnetic fields there.

Observationally, there are three characteristics of solar oscillations that make them wonderful probes of physical conditions in the Sun: (1) there are many modes (some 10^7), each carrying somewhat different information about the interior, (2) the modes are weakly damped, lasting thousands of cycles each, so their properties can be measured accurately, and (3) the amplitudes are small enough (highest amplitudes are 10^{-7}, with Mach numbers

10^{-4}) to be treated in the linear regime, greatly simplifying theoretical analysis and interpretation (e.g., back reaction needn't be considered).

Ask class: for the Sun and for other stars, how would we observe these oscillations? By photometric variation or by Doppler shifting of lines, or both. For the Sun we have the option of doing 2-D Doppler imaging of the surface, and hence getting a lot of information. Most of the oscillation modes in the Sun have high horizontal wavenumbers. **Ask class:** why is it that this makes detection of such modes extremely difficult for stars other than the Sun? Because we usually can't resolve the disk of other stars, and hence averaging happens.

Terminology: traditionally, oscillation modes are characterized by the restoring force responsible for the oscillations. The two classes are p-modes, in which pressure forces are the restoring forces, and g-modes, in which it is gravity. In the Sun, the oscillations are mainly sound waves, and hence are p-modes. In white dwarfs, buoyancy is the restoring force, so these are typically g-modes. There are some pulsation modes that seem to straddle categories, looking like p-modes in the envelope but g-modes in the core.

With respect to fluid motions in stars, we've considered two extremes. One is that there is no motion; this is the basis of hydrostatic equilibrium. The other is that there is an explosion (e.g., a supernova). Now we'll consider an intermediate possibility: the star is oscillating, but in a way such that the fluid elements are *almost* at rest. We can therefore treat motion as a perturbation. The general strategy is to linearize the equations of stellar structure and to assume that the perturbations can be written as the sum of separate normal modes, each of which is the product of a spatial part and a temporal part. The equations can then be solved for each mode independently. To start, let's see if we can approach a more simplified problem.

Ask class: typically, how do dynamic timescales and thermal timescales compare to each other? Dynamic timescales are *much* faster, so an individual oscillation essentially happens in an adiabatic approximation, where the total energy content of any given fluid element is unchanged. **Ask class:** what about the long term? Suppose a particular oscillation is perfectly periodic; what can we say about the energy content of the star over a long time? It must remain constant, otherwise the motion would *not* be strictly periodic. In that case, the adiabatic approximation is even better. Therefore, we will ignore *all* heat exchange mechanisms. This reduces the star to a mechanical system without heat losses, a bit like sound waves in a box. **Ask class:** given this approximation, which of the four fundamental equations of stellar structure survive? Just the mass and hydrostatic equations, except that now we're allowing some motion. Then

$$\frac{\partial M_r}{\partial r} = 4\pi r^2 \rho \tag{2}$$

and

$$\ddot{r} = -4\pi r^2 \left(\frac{\partial P}{\partial M_r}\right) - \frac{GM_r}{r^2} \ . \tag{3}$$

Perfect equilibrium, of course, means $\ddot{r} = 0$, in which case we recover the hydrostatic equation.

Now the point is to apply a perturbation analysis. We want to use a Lagrangian description, and hence consider departures of the radius and density, as a function of time and mass shell M_r, from their equilibrium positions. **Ask class:** how should we proceed? By assuming that the radius and density are their equilibrium values plus a small perturbation:

$$r(t, M_r) = r_0(M_r)\,[1 + \delta r(t, M_r)/r_0(M_r)] \tag{4}$$

$$\rho(t, M_r) = \rho_0(M_r)\,[1 + \delta\rho(t, M_r)/\rho_0(M_r)] \tag{5}$$

Ask class: what's the next step in the perturbation analysis? Put these quantities into our two equations, carry out expansions to linear order, and subtract out the zeroth order equations. Let's do an example, for the mass equation. **Ask class to lead through:**

$$\frac{\partial M_r}{\partial[r_0(1 + \delta r/r_0)]} = 4\pi[r_0(1 + \delta r/r_0)]^2[\rho_0(1 + \delta\rho/\rho_0)] \ . \tag{6}$$

Ask class: what is the result when we take the partial through the denominator? Get $(1 + \delta r/r_0)\partial r_0 + r_0\partial(\delta r/r_0)$. Take out the factor $\partial M_r/\partial r_0$ from this side of the equation, invert and take the first order terms. Then, expand right hand side to first order. **Ask class:** what is the result?

$$\frac{\partial M_r}{\partial r_0}\left[1 - \frac{\delta r}{r_0} - r_0\frac{\partial(\delta r/r_0)}{\partial r_0}\right] = 4\pi r_0^2\rho_0\left(1 + 2\frac{\delta r}{r_0} + \frac{\delta\rho}{\rho_0}\right) \ . \tag{7}$$

Whew! We know that the dominant terms (the "1"s on both sides) cancel each other because $\partial M_r/\partial r_0 = 4\pi r_0^2\rho_0$ from the equilibrium (zeroth order) equation. So, we divide out those factors and cancel the 1s, giving finally

$$\frac{\delta\rho}{\rho_0} = -3\frac{\delta r}{r_0} - r_0\frac{\partial(\delta r/r_0)}{\partial r_0} \ . \tag{8}$$

The next step (also standard in perturbation analyses) is to assume that all perturbations can be decomposed into Fourier components, in which the time variation is represented by exponentials. For example,

$$\frac{\partial r}{r_0}(t, r_0) = \zeta(r_0)e^{i\sigma t} \ . \tag{9}$$

If the perturbation is adiabatic, we can close the equation set by using the relation $(\partial \ln P/\partial \ln \rho)_{\rm ad} = \Gamma_1$ between pressure and density perturbations.

Now, if we were to write the mode more generally (not assuming spherical symmetry any more), we would have

$$\zeta_{nlm}(r, \theta, \phi, t) = \zeta_{nl}(r) Y_l^m(\theta, \phi) e^{-i\omega_{nlm} t} \ . \tag{10}$$

Here, n is the radial order (number of nodes in the eigenfunction between the center and surface), l is the angular degree (product of the stellar radius and total horzontal wavenumber), and m is the azimuthal order (projection of l on the stellar equator, therefore $m \leq l$). For a spherically symmetric star, m doesn't matter. Anything breaking that symmetry (e.g., rotation or magnetic fields) breaks the degeneracy and splits modes. Observationally, we can see frequencies, amplitudes, and linewidths; frequencies, which can be measured most accurately, usually contain the most information.

A lot of the information from these modes (e.g., about rotation in the Sun) require high-l modes. Some doesn't: for example, some of the astrophysical solutions proposed for the neutrino deficit (cool center, mixing of fuel) don't match the observed frequency separations.

Other stars also have detectable pulsations. A notable example is white dwarfs (oddly enough, this was detected in 1968 by Landolt, whose purpose was to establish photometric standards, since it was believed at the time that white dwarfs were rock-solid in their luminosity!). These stars have much higher amplitude oscillations (mostly g-mode), and from some of them, observations using the Whole Earth Telescope (a set of telescopes around the world allowing many days of continuous coverage) have revealed more than 100 separate modes. These provide information about rotation, magnetic fields, masses, and so on that is obtainable in no other way. There is, however, a *lot* to be done. We're in a strange situation, in that there are thousands of times more modes observed on the Sun than in all other sources combined; our information is therefore rather lopsided, so much theory and observation has to improve before this technique can be used more generally.