Reference: De Loore, C.W.H., Doom, C. Vol.179 - Structure and Evolution of Single and Binary Stars. 1992. QB 1 .A32D7 – ASSL Series.

For report on paper, I need the reference or references by beginning of class on Thursday, April 25. The written reports are due on Thursday, May 2. There will be seven presentations in class on Tuesday, May 7 and seven on Thursday, May 9. Each presentation will be 8+2; that is, 8 minutes for the talk and 2 minutes allowed for questions afterwards. I will keep time strictly. For the in-class presentations, the guidelines are on the class web page and were handed out at the beginning of class. The order of presentations will be chosen by random drawing on May 2.

I will treat the speakers of a given day to dinner, and will provide munchies during the talks :).

**The Hayashi track**

Young low-mass stars are fully convective. HK say this is because their luminosities are high and their temperatures are low. **Ask class:** why does full convection follow from this? Because low temperature implies high opacities, which along with high luminosities means convection. Full convection turns out (Hayashi 1961) to mean that their tracks are at constant effective temperature $T_{\rm eff}$, so their tracks are vertical in the H-R diagram. There is a forbidden region for lower temperatures: if the temperature were lower than this limit while fully convective, the star would not be in virial equilibrium. See HK 7.3.3 for lots of messy algebraic details.

Near verticality is due to H- opacity being so temperature-sensitive (also reason why red giant branch is nearly vertical). From HK equation (7.161), with a temperature coefficient corrected below the equation, the effective temperature on this branch is

$$T_{\rm eff} \approx 4000\mu^{13/51}\left(\frac{M}{M_\odot}\right)^{7/51}(L/L_\odot)^{1/102}\ . \qquad (1)$$

This means that on this track, the effective temperature is virtually independent of the luminosity. **Ask class:** what physical effects mean that a star isn't always on this track? Accretion will boost the temperature to about 10,000 K (because that's the sweet spot in the cooling curve), which will destroy $H^-$. Thermonuclear fusion will increase the temperature, and the luminosity will be smaller, so for most main sequence stars radiation can carry energy instead of requiring convection.

Hayashi track is mainly a theoretical construct instead of an observational fact. Most of the track, if it exists, is not visible because of dust obscuration and D burning sequence. If there is accretion, it stays at about 10,000 K (because that's the sweet spot in the cooling curve) which is enough to destroy H-.

In principle, placing a PMS star on the H-R diagram will indicate its mass and age. In practice, it is very difficult to make accurate models of PMS stars. One reason is that convection is dominant, and mixing-length theory is not exactly precise! Radiative opacities are also rough, because of the contributions of molecules and dust/grains. This is especially challenging for low-mass stars.

**Evolution of Solitary Stars**

We now move to the evolution of single stars, i.e., stars not in close binaries or multiple systems (we'll get to those later). The evolution of such a star depends primarily on its mass, but this is because of the microphysics involved. Here are some framing questions to determine the course of evolution:

(1) When the star burns hydrogen on the main sequence, does it do so via the pp chain or the CNO cycle? **Ask class:** which is more important for high-mass stars? Why? CNO, because high-mass stars have higher temperatures in the center (for hydrostatic, virial equilibrium), and CNO energy rate is highly temperature-sensitive.

(2) When the star starts to burn helium in the core, is the helium degenerate or nondegenerate? **Ask class:** they did a problem on this; when happens when helium ignites when it is degenerate? Helium flash, rapid burning. **Ask class:** why is there a fundamental difference between ignition while degenerate vs. nondegenerate? Nondegenerate will expand and cool; degenerate won't, so there can be a runaway.

(3) When/if the star starts to burn CO in the core, is the CO degenerate or nondegenerate?

(4) Is the metallicity that of solar-type stars (Pop I) or is it much lower (Pop II)?

(5) At a given stage of evolution, is mass loss important for the structure of the star? **Ask class:** what could cause mass loss? Radiative driving. **Ask class:** would this be important for high or low luminosity stars? Only high luminosity produces enough radiation force (line driving is important).

These questions determine the pace and nature of the evolution of stars, which is especially well-represented in the Hertzprung-Russell diagram. We'll start with low-mass stars. This lecture is the topic of Chapter 2 in HK, but I'm following the more extensive treatment by de Loore and Doom.

**Evolution of low-mass stars**

Key points:

(1) Helium core burning happens while core is degenerate, leading to a flash. **Ask class:** why true for low-mass stars? Core is cooler, so it has a better chance of being degenerate.

(2) Stars below $1.2\,M_\odot$ burn by pp, which generates little energy and does not require convection for transport, so core is not convective.

(3) Stars above $1.2\,M_\odot$ burn by CNO, which is more efficient and therefore does require convection. Their cores are convective. Note that part of this is the large *gradient* of energy generation, due to the sharp temperature sensitivity.

(4) Conversion of four hydrogen into one helium reduces the number of particles, so for ideal gas pressure to be maintained the density (g/cc) and temperature have to go up, so the core contracts. **Ask class:** suppose we start with fully ionized pure hydrogen, and generate fully ionized pure helium. What is the factor by which the number of particles changes? Four H (p+e) have eight particles, 1 He (alpha+2 e) has three particles, so 8/3. This is often phrased in terms of mean molecular weight. **Ask class:** what is the mean molecular weight of fully ionized pure hydrogen? Fully ionized pure helium? Mean molecular weight is just the total mass divided by the total number of particles. If it is expressed as a dimensionless number, it's this average mass divided by the mass of a nucleon. So, pure ionized H=1/2, pure ionized He=4/3. Same factor of 8/3.

(5) This contraction and heating increases the nuclear reaction rate and therefore increases the luminosity. This process accelerates with time.

(6) From the energy transfer equation,

$$L_r = -\frac{4\pi r^2 c}{3\kappa\rho}\frac{d(aT^4)}{dr}\ , \tag{2}$$

an increased luminosity means an increased radius, since the remaining quantities cannot change dramatically. Therefore, as the core contracts, the stellar radius expands.

(7) At some point the hydrogen in the central regions is exhausted. This happens gradually for non-convective cores ($M < 1.2\,M_\odot$), and more suddenly for convective cores. **Ask class:** why more sudden for convective cores? Because composition is mixed then.

(8) This causes different behavior in the HR diagram for different masses (show). When the whole convective core exhausts its hydrogen, the whole star falls short of energy and contracts. Therefore, the radius decreases slightly, the surface temperature increases, and the star moves to the left in the H-R diagram.

(9) This motion stops only when all hydrogen in the central core is depleted. The star then enters the phase of *hydrogen shell burning.*

(10) The central temperature is not yet high enough to burn helium, but the temperature outside the core is enough to burn hydrogen in a shell.

(11) Node theorem. When there is burning in a shell, it tends to be at a node of the movement of matter (that is, it doesn't move in or out radially much even if other parts of

the star are expanding or contracting). That's because if it moves out it cools and stops burning, causing it to fall inwards to where it is hot enough to burn; if it moves in it heats up so much that it expands. Therefore, if matter is contracting inside a shell it is expanding outside, and vice versa. In a star with two shells (as can happen with a helium shell close to the core, and a hydrogen shell farther away), if the core contracts then so does the outer envelope, and if the core expands then so does the outer envelope.

(12) The inert helium core therefore grows, the burning shell moves out, and the star moves towards the giant branch. For $M < 1.2\,M_\odot$ this happens gradually.

(13) For $M > 1.2\,M_\odot$, the central helium core is larger, the core contraction happens faster, and lots of gravitational energy is released. This happens so fast that the expanding layers cool, and the opacities of the outer layers increase and become opaque. **Ask class:** if $\kappa$ goes up, what has to happen to the radius for a given luminosity? The radius has to increase even more, giving stronger cooling, increasing opacities. This is a runaway and produces a very rapid expansion. It is halted when the envelope becomes fully convective. At that point, the star ascends the giant branch.

(14) The contraction of the helium core increases its density without increasing its temperature much, and for low-mass stars the helium core becomes degenerate.

(15) When the central temperature becomes $T > 10^8$ K, the triple-alpha process operates, releasing energy. This runs away because the core cannot cool by expansion (since it's degenerate). This helium flash continues until the temperature is high enough to lift degeneracy, at which point $\sim 20\%$ of the helium has probably been consumed. The helium flash is thought to last a few minutes. **Ask class:** would we expect to see an observational signature of the flash? No, because the time necessary to transport the energy is millions of years.

(16) After the flash, $\sim 10^6$ yr are required to stabilize the star, after which helium burning happens quietly. **Ask class:** would we expect the helium burning phase to be longer or shorter than hydrogen? Shorter, because less energy is available and because the energy generation rate is higher. Get recap of previous situation, with an outer H burning shell, inert He, a He burning shell, and degenerate CO in the core.

(17) This leads to thermal pulses. Luminosity in the He burning shell increases, expanding and cooling the envelope, shutting off the H shell burning, decreasing the luminosity, contracting and heating the envelope, reigniting the H shell, and so on. These pulses occur with a $\sim 10^5$ yr interval between them, and the luminosity can change by a factor of 2. After one full cycle, the luminosity has gone up a little, but the high temperature sensitivity of the opacity means that the temperature of the surface is about constant, so the star ascends a roughly vertical track (similar to the protostellar Hayashi track).

(18) Dynamical instability. Near the end of the AGB, luminosity can increase by a factor 10 before rapidly decreasing, and the size can change by 50%. Mira Ceti, the prototype of Mira variables. Lots of mass lost during this phase: 20–30%. Finally, it reaches a point where it is no longer unstable (when enough H and He have been consumed), and it shrinks into a white dwarf. The ejected mass is illuminated by the hot central star, and appears as a planetary nebula.

(19) Low-mass, low-metallicity (population II) stars. **Ask class:** if there are few metals, what effect will this have on opacity? It will lower the opacity, meaning that for all else equal a star with smaller metallicity is more compact and its surface temperature is higher. This led to one of the big revisions in the distance scale from Cepheids: the period-luminosity relation differs from Pop I to Pop II, and as Pop II are brighter for a given period, they are farther away for a given flux.

(20) After the He flash in Pop II stars, they shrink more rapidly than Pop I and therefore have a roughly constant luminosity with an increasing temperature: this causes motion to the left in the H-R diagram, and produces the *horizontal branch*. In the middle of this branch the stars are unstable to pulsations, producing the RR Lyrae variables. This is an opacity variation in the layers where H, He are partially ionized (around $10^4$ K).

(21) Their final stage is as white dwarfs, which simply cool (we'll talk more about the structure of white dwarfs later). **Ask class:** given that WD have no energy sources, and simply cool off, what is their general path in the HR diagram? From upper left (hot and luminous) to lower right (cool and dim). **Mention:** analytic treatments of white dwarf cooling and luminosity as a function of time have been compared to the observed luminosity function of white dwarfs in an attempt to estimate the age of the Galactic disk. Lots of subtleties, but $10^{10}$ yr is a good estimate.

(22) About 25% of all stars that have a helium flash are expected to become WD with no detectable hydrogen on the surface (non-DA white dwarf). The other 75% do have detectable hydrogen (DA white dwarf).