Convection

Chapter 5 in HK.

Focus on aspects of mixing length theory. Present picture first, then general ideas, then more specifics. At the end, students should have an idea of how to calculate with it, but also a solid appreciation of its strengths and weaknesses.

We are now going to tackle the third major type of energy transport, convection. There are aspects of this that differ significantly from our treatment of radiative and conductive energy transport. In those cases it might be difficult in practice to calculate a particular opacity, but in principle it's fairly clear: since microscopic quantum mechanics underlies all of it, one "simply" calculates the equilibrium properties of atoms and electrons for a given set of local conditions.

Convection is different. The general process of convection has some fluid element rising or falling and dissipating its energy as it does so. In its full generality, therefore, convection is a *nonlocal* process and involves very poorly determined details of fluid mechanics. Alas, it is also exceedingly important for most stars, especially the low-mass ones.

As a way around this, "mixing length theory" has been developed. It has the advantage that it mocks up a local treatment of convection, and often doesn't do too badly for real stars. Unfortunately, in stars like the Sun it violates its own assumptions and is laced through with estimates and approximations. Convection is still a major problem in stellar models, and no clear alternative to mixing length theory has arisen yet.

General idea of convection

(draw on board) Suppose you have some element of fluid at a higher temperature than the surroundings. **Ask class:** for pressure balance with its surroundings, what does this mean about the density? Density is less, so buoyancy effects cause it to rise. Now it's up further, so the density and pressure of the surrounding medium is less. **Ask class:** if the pressure is less, what happens to the fluid element? It expands until the pressure is in equilibrium with its surroundings. So, **Ask class:** what happens to its density? The density drops. **Ask class:** what is the condition for the element to keep rising? The new, adjusted density needs to still be less than the density of the surroundings, so that buoyancy effects continue to cause the fluid element to ascend. **Ask class:** what does that mean about the new temperature of the element compared to the temperature of its new surroundings? It means that the temperature of the element has to be greater than the temperature of the surroundings.

Now let's think about what this means for energy transport. Suppose that the fluid element rises much more slowly than the speed of sound, so that pressure balance is maintained, but much more rapidly than the time necessary to have heat leak out of (or into) the fluid element. Then the total heat in the fluid element is conserved, and if we can ignore viscosity (which we usually can for this purpose), it means that the element moves adiabatically. This means that the entropy is conserved, so that the temperature gradient is fixed for a given pressure gradient. Call this gradient $\nabla T_{\rm ad}$. Thus, for a given fluid element with an initially small perturbation, we know how its temperature will change as it rises.

Given this, **Ask class:** what is the condition on the gradient of the temperature of the surrounding medium such that the fluid element will continue to rise once perturbed upwards? Since the surrounding temperature has to continue to be smaller than the temperature of the fluid element, it has to drop with increasing height faster than the temperature of the element drops. Therefore, the temperature gradient ∇T must satisfy $\nabla T > \nabla T_{\rm ad}$. This is the Schwarzschild criterion for convection, brought to you by the same person who came up with the Schwarzschild spacetime for uncharged, nonrotating black holes.

If this criterion is satisfied, it implies large-scale motion of fluid. In addition to hot parcels of fluid rising, cold parcels will drop, so there is a net transfer of energy from the hot stuff below to the cold stuff above. Note that the macroscopic motions mean that concepts such as mean free path are a bit dicey, compared to their rather clearer analogues for radiative or conductive energy transport. There are also complicating effects in reality. For example, heat really can leak into or out of the fluid element as it rises or sinks. Before addressing issues like that, though, let's hold on a bit and develop our general intuition about convection some more.

People frequently determine whether a given layer of fluid is convectively unstable by calculating its structure without including convection, then applying the Schwarzschild criterion or an alternative. Let's assume that for the layers of interest, all the energy was generated at a much deeper layer. Given that large temperature gradients are needed for convection, **Ask class:** what does this imply about other opacities? The amount of energy to be transported is fixed by the deeper layers, so large temperature gradients would have to mean high opacities. **Ask class:** do they expect white dwarfs or neutron stars to have convective layers under ordinary conditions? No, because for these stars conduction is extremely effective, so the temperature gradient is tiny and convection is suppressed. Only when other forms of energy transport are ineffective does convection come to the rescue. This, by the way, is one reason why we can often ignore the diffusion of energy out of our fluid elements: for convection to be important at all, this diffusion needs to be relatively weak. In main sequence stars it tends to be in the somewhat cooler layers (where line and edge opacities are important) that convection occurs. Low mass stars have large convective zone for this reason, with the interesting consequence that dynamos generate pretty strong magnetic fields, implying lots of magnetic activity. Higher-mass stars have fewer sources of opacity and therefore have relatively weaker fields.

So how do we put this in to a description of energy transport? The most widespread model is mixing length theory. In mixing length theory the assumption is that the fluid element rises some distance ℓ, then releases its energy (i.e., comes into thermal equilibrium with the new environment). The reverse process can also happen: a cool fluid element, which is denser than average because of pressure balance, sinks some distance ℓ and gains energy from the environment. In both cases, energy is transported outward. Of course, the fluid element can "leak" energy on its way up or down as well.

This is the core of mixing length theory. The amount of energy that it transported depends on ℓ, the buoyant velocity w, the heat content of the star as a function of depth, and how much energy drifts out of the elements as they rise or fall. These quantities can be computed at a single radius, making MLT a "local" theory and therefore useful for computation.

Assumptions of MLT

These assumptions constitute the "Boussinesq" approximation: (keep these on the board).

1. A readily identifiable fluid element has a dimension comparable to ℓ.

2. The mixing length is much shorter than any other scale length associated with the star (e.g., pressure, temperature scale heights). *Violated* for the sun, where the pressure scale height is comparable to what is needed for ℓ!

3. The fluid element is always in pressure equilibrium with its surroundings. This means, for example, that ℓ/v_s is much shorter than ascent or descent times.

4. Acoustical phenomena, shocks, etc., may be ignored.

5. The density and temperature of the fluid element deviate only slightly from the environment.

Together, these assumptions mean that the fluid is almost incompressible and that density and temperature variations are small. Note that assumption 2 is often violated, and that assumption 1 is troublesome by itself: if the fluid element is the same size as ℓ, it is tough to picture the slow drift and leakage of energy!

Another problem is that lab experiments can't simulate the conditions in stars, in the sense that some of the dimensionless numbers that characterize the fluid are a factor of a billion or so different between stars and laboratories. However, note also that this is a common situation in astrophysics. What you have to do in these circumstances is do the best you can, make reasonable assumptions, and solve a simpler problem. It would, of course, make no sense to go wild with this and end up with a finely refined variant of MLT when you know that some of the initial assumptions are suspect!

Ask class: suppose that convection occurs. What does this do to the temperature gradient? It tends to reduce the temperature gradient, by putting hot fluid higher and cold fluid lower. **Ask class:** if we were to try to incorporate convection into the same general opacity scheme as we used for radiation and conduction, would we include it linearly or harmonically? Harmonically, because as another channel for energy transport it increases the flux and decreases the effective opacity.

Ask class: suppose that convection is extremely efficient. To a first approximation, what is the temperate gradient you'd end up with after convection had changed the gradient? It will be very close to, but slightly larger than, the adiabatic gradient. It has to be larger to continue convection, but if convection is efficient then ∇T doesn't have to be much larger than $\nabla T_{\rm ad}$ to transport lots of energy.

As indicated in the previous notes and in the book, the flux due to efficient convection is

$$F_{\rm conv} = \rho w c_P \Delta T\ . \tag{1}$$

Here a parcel of energy is typically going at a speed w, the typical temperature contrast with the surroundings is ΔT, ρ is the density of fluid and c_P is the specific heat at constant pressure (energy per mass per temperature). How can we test if this is reasonable? The units work (can verify). It makes sense that if there is no temperature contrast ($\Delta T = 0$) there will be no flux. The higher the speed the larger the flux, which is right. If the specific heat is low, little energy can be transported, so F should depend directly on c_P as it does. Finally, if there is more matter (high ρ), more flux is transported.

More details, including radiative leakage, buoyancy, and details about how to compute radiative flux, can be found in the year 2000 notes and in the book. Suffice it to say that although mixing length theory does a decent job, full convection is a tough nut to crack. There are a number of attempts to do better than MLT; overshooting, for example. "Correct" treatment would require full fluid (MHD in general) equations, and is very difficult and complicated. When we get to modeling stars, we'll see a bit more how to include convection in reality.