What do we know about stars?

Ask class: for suggestions on each of the following lists.

Masses, luminosities, colors, lifetimes and eventual fate, rotation. Single, binary, multiple, formation, pulsation, magnetic fields. Flaring, composition, winds, jets.

What do we really want to explain about stars?

Mass range, luminosities, temperatures, ages, composition. What makes them shine, what holds them up, what state is matter in.

How do we know these quantities from observations?

Major points: distances and ages. Must know distances to get luminosities. Age $\gg$ human lifetime, so need another way to see them evolve. Go through how this is done.

This is going to form the framework of the course. We will place ourselves in the position of talented physicists who want to work out the structure and evolution of the stars. That means that we will not only have to come up with answers to questions, but we need to come up with the questions themselves. Given the phenomenal inherent complexity of stars, we have to choose carefully what problems we are studying. These have to be problems that are simplified enough to be analyzed, but realistic enough to give us insight and to be compared with observations.

What simplifying approximations can we make?

First, what can we ignore? "Ignore" here means that what we are neglecting is small enough compared to what we are including that the accuracy we need is satisfied. Thus, need in every case to have some standard against which to compare.

Ignore rotation.—Start with the case of the Sun. The Sun has a rotation period of about a month. The Keplerian period at the surface is $2\pi\sqrt{R^3/GM}$, or about 3 hours. Thus, the rotation of the Sun is about 1/250 of Keplerian, and effects should be quadratic in that, so negligible at the 10^{-4} level.

Ignore magnetic fields.—Strongest fields seen are in sunspots, $B \sim 1000$ G. If that were the average in the entire Sun, we'd have a total energy of $(B^2/8\pi)\frac{4\pi}{3}R^3 = 6 \times 10^{37}$ erg. Compared with the $GM^2/R = 4 \times 10^{48}$ erg in gravitational potential energy, this is tiny and can be neglected.

Assume spherical symmetry.—This follows from the lack of rotation. To a high degree of accuracy the Sun is spherically symmetric.

Assume that the Sun has cosmic composition?—Actually, we can get information about this that doesn't require an assumption. That is, we can look at the composition of the solar photosphere, we can see what is in meteorites, and so on. For some approximations we can assume that the Sun is pure hydrogen, but actually for much of the development we don't need to worry about composition at all.

Ask class: how much has the Sun changed in their lifetimes?

Assume that the Sun is in equilibrium.—Ah, but this requires more development.

What does the condition of equilibrium mean?

Observationally, the Sun more or less stays the same in many important ways. That is, its radius is essentially constant and its luminosity is essentially constant, among other things. Need to quantify this and represent it with appropriately simple equations.

Dynamic, or hydrostatic, equilibrium.—This means that the star as a whole stays put. Said another way, the forces acting on any given parcel of gas balance each other. First **Ask class:** what would happen if the Sun were far away from this balance? Answer: it would collapse or expand on the dynamic time scale, which to a decent approximation is just the free-fall time scale. That, in turn, is roughly $1/\sqrt{GM/R^3}$, or about $1/\sqrt{G\rho}$. For the Sun, the average density ρ is about 1 g cm^{-3}, so that's 1 hour. Since we don't see dramatic changes in the Sun on 1 hour time scales, and indeed not on scales of millions of years (**Ask class:** how do we know? Fossil, geologic record.), we know that this overall balance holds to extreme accuracy. Not a good approximation for supernovae, of course, but even for most pulsators the bulk of the star is in hydrostatic equilibrium.

Now need to quantify what is balancing what. For a given parcel of gas, gravity pulls down. **Ask class:** what could oppose gravity? If the pressure *gradient* (not just the pressure) is in the same direction of gravity (i.e., more pressure farther down), then this opposes gravity. Let's say that we have a parcel of gas with area perpendicular to $\hat{r}$ of A and thickness dr. If the density is ρ, then the gravitational force on this is $-GM\rho Adr/r^2$, where the negative sign indicates a downward force. This can be written as $g\rho Adr$, where $g = -GM/r^2$ should really be a vector, and of course M is really M_r, the mass interior to r. The force due to the pressure gradient is $P(r)A - P(r+dr)A = dr(dP/dr)A$. The sum of the two has to be zero for force balance, so $dr(dP/dr)A + g\rho Adr = 0$, or finally $dP/dr = -\rho g$ (more generally, when spherical symmetry doesn't apply, $\nabla P = -\rho\mathbf{g}$). **Ask class:** does this have the right units, limits? This is the equation of hydrostatic equilibrium, and is one of the four fundamental equations of stellar structure. **Ask class:** how would this be modified if a star were rotating rapidly? Have to include centrifugal terms. In general, need force balance for dynamic equilibrium. In the case of stars (and most other things in the universe), this translates to gravity vs. everything else, because gravity is universally attractive and hence other forces are needed to balance it.

It is often helpful to write such equations in terms of the mass instead of the radius. This formulation, in which we follow the mass, is called the *Lagrangian* formulation. Then, since the mass in a spherical shell is $dM_r = 4\pi r^2 \rho dr$, the equation of hydrostatic equilibrium is $dP/dM_r = -GM_r/4\pi r^4$.

Thermal equilibrium.—**Ask class:** Is the Sun in thermal equilibrium? Of course not! It has a surface temperature of almost 6000 K, and it is radiating into cold, empty space. So what does it mean to say it is in thermal equilibrium? What about deeper in the Sun? What is the microscopic condition for something to be pretty close to thermal equilibrium? One criterion could be that whatever is carrying the energy can't go so far that it samples temperatures that are dramatically different from where it started. In the case of the Sun, we know that the whole temperature run is about 10^7 K in 10^{11} cm, or about 10^{-4} K cm^{-1}. As we'll find out later, photons carry most of the energy. The cross section is no less than about 10^{-24} cm^2 (the Thomson scattering cross section). At 1 g cm^{-3}, the average for the Sun, that's a number density of 10^{24} cm^{-3}, implying that the average photon travels 1 cm. It therefore samples an average temperature change of 10^{-4} K; since the temperature even at the photosphere is 6000 K, this is a tiny fraction and thermal equilibrium is a good approximation for almost all of the Sun, even where the density is much smaller than the average. Might be reasonable to discuss here vertical versus lateral temperature gradients on the Earth. This temperature gradient implies 10 K in a kilometer. This is not ridiculous for what you'd get if you went up in the atmosphere, but it is ridiculous for a lateral temperature gradient (the temperature would have to change by 100 K when you went to D.C.!).

Thermal balance.—Ah, but there is another issue. Energy isn't just leaking out of the Sun. It is being generated as well. For each parcel of fluid the energy generated must flow out at the same rate it is generated, or there will be a buildup of heat. Thinking again about a parcel of area A and thickness dr, and assuming that the energy generation rate per mass is ϵ and the flux at r is $F(r)$, we have $F(r+dr)A - F(r)A = \epsilon\rho A dr$. In this expression we can replace A by $4\pi r^2$, the area of the spherical shell at r, and define the luminosity at r as $L(r) = 4\pi r^2 F(r)$. Then $L(r+dr) - L(r) = \epsilon\rho 4\pi r^2 dr$, or $dL/dr = 4\pi r^2 \rho\epsilon$. This is the equation of thermal balance. In the Lagrangian formulation this is $dL/dM_r = \epsilon$.

Minimum energy.—The total energy of the star must be a minimum given the constraints, or it will quickly go to the real minimum. See if they can guess what this will lead to. Refer them to H+K derivation, quote result of variation. **Ask class:** what should the first variation be? Zero, because it's an extremum. Here, show them how one gets the equation of hydrostatic equilibrium by variational arguments. **Ask class:** what should the second variation be? Positive, because it's a stable equilibrium (a minimum). Note that the process of going to equilibrium is related to protostars.