Practice With Addition of Opacities

1. Consider a pure hydrogen plasma of temperature T. Assume that there is a source of radiation propagating through the plasma, and that the *initial* spectrum (prior to being affected by the plasma) is that of a blackbody, also at temperature T. The sources of opacity that we will consider are bound-free, free-free, and Thomson opacity. The fundamental cross sections of the processes for a photon of energy $\hbar\omega$ are

$$\begin{aligned}
\sigma_{\rm b-f} &= 2\times 10^{-14}(\hbar\omega/1\,{\rm eV})^{-3}\ {\rm cm}^2 \\
\sigma_{\rm f-f} &= 2.6\times 10^{-35}(\hbar\omega/1\,{\rm eV})^{-3}(n_e/1\,{\rm cm}^{-3})(T/1\,{\rm K})^{-1/2}\ {\rm cm}^2 \\
\sigma_{\rm T} &= 6.65\times 10^{-25}\ {\rm cm}^2\ .
\end{aligned} \tag{1}$$

Note that n_e is the number density of *free* electrons, not of all electrons. For the problems below, remember that a conversion between temperature and energy is $1\,{\rm eV} = 1.16\times 10^4$ K.

(a) Suppose the plasma is 100% neutral, has a temperature of 3×10^6 K, and a number density of 10^{20} cm^{-3}. Which of the three opacities is most important in determining the transport of radiation?

(b) Suppose the plasma is 100% ionized, has a temperature of 3×10^6 K, and a number density of 10^{20} cm^{-3}. Which of the three opacities is most important in determining the transport of radiation?

(c) Suppose the plasma is 50% ionized, has a temperature of 3×10^6 K, and a number density of 10^{20} cm^{-3}. Which of the three opacities is most important in determining the transport of radiation?

The spectrum may be distorted by propagation through the plasma. Qualitatively, how is it distorted in each case?

2. Consider again a pure hydrogen plasma, of temperature T and mass density ρ g cm^{-3}. Suppose that the Rosseland mean opacity (total radiative opacity, correctly averaged over photon frequency, weighted by the spectrum, and including all radiative opacity sources) is approximately

$$\kappa_R \approx 10^{23}\rho T^{-7/2}\,{\rm cm}^2\,{\rm g}^{-1}\ , \tag{2}$$

where ρ is the density in units of 1 g cm^{-3} and T is the temperature in Kelvin. WARNING: this is not really correct, it's just a simplification! Conduction may also take place; in conduction, the energy is carried by electrons instead of photons. Let's say that the conductive opacity, weighted over the thermal electron distribution, is

$$\kappa_{\rm cond} \approx 4\times 10^{-8}T^2\rho^{-2}\,{\rm cm}^2\,{\rm g}^{-1}\ . \tag{3}$$

(a) Suppose that $T = 10^6$ K and $\rho = 100$ g cm^{-3}. Is it radiation or conduction that primarily determines how energy is transported through this plasma?

(b) Suppose that $T = 10^7$ K and $\rho = 100$ g cm^{-3}. Is it radiation or conduction that primarily determines how energy is transported through this plasma?