ASTR 601 Problem Set 1: Due Thursday, September 18

1. (4 points total)

In order to get full credit for this problem you need to assess *each* of the possible solutions. That is, you need to determine whether each of the solutions *could* be correct. If you find something definitely wrong with a solution you can move on to the next possibility, but just passing a single test is not enough.

A turbulent fluid emits sound waves. If the turbulent velocity is v, the sound speed in the fluid is c_s, and the length over which turbulent fluctuation velocities is correlated is l, what is the energy per time ϵ_s emitted as sound by unit mass of the turbulent fluid?

A) $\epsilon_s \propto c_s^6/(v^3 l)$
B) $\epsilon_s \propto c_s^2 v l$
C) $\epsilon_s \propto v^8/(c_s^5 l)$
D) $\epsilon_s \propto v^3/l$

2. (4 points total) A nonrotating neutron star has a circumferential radius R measured locally ("circumferential" means that you measure the circumference at the equator, and then divide by 2π). The star also has a gravitational mass M ("gravitational mass" means the mass that you would measure using, e.g., Kepler's laws at a large distance from the neutron star). The gravitational redshift z of a photon of any energy from the star is given by $1+z = 1/\sqrt{1-2GM/(Rc^2)}$, where G is Newton's gravitational constant and c is the speed of light in a vacuum. The star has uniform blackbody radiation from its surface, emitted isotropically and with temperature $T_{\rm surf}$ as measured locally on the surface. You, the observer, are at a distance d from the neutron star; d is finite but is so much larger than R that you are effectively in flat spacetime with no gravitational redshift due to the star. Given this setup, and using your understanding of specific intensity:

(a) (1 point) Derive the blackbody temperature that you measure from the star.

(b) (3 points) Derive the angular radius that you measure for the star.

3. (4 points total) Dr. I. M. N. Sane, an independent physicist, has realized that random walks almost never apply in astronomy and are therefore useless. He gives as an example that very high-energy photons have a much higher probability of forward scattering (scattering in their direction of motion) than backward scattering (scattering opposite to their direction of motion). He has submitted a manuscript to this effect to Nature magazine. Urmila Chadayammuri, astronomy editor at Nature, has contacted you to ask for your opinion.

In particular, Dr. Sane considers a case in which a photon will scatter uniformly anywhere between 0 radians and $\Delta\theta \ll 1$ radians from its original direction. The direction of the deviation is uniform: if the photon was originally traveling in the z direction ($\theta = 0$), then after the scattering, θ has an equal probability of being anywhere between 0 and $\Delta\theta$, and ϕ (the azimuthal angle) has an equal probability of being anywhere from 0 to 2π. After that scattering, the next scattering is as described, but relative to the direction into which the photon scattered in the previous step. For example, if the original direction was $\theta = 0, \phi = 0$ and it scattered into a direction $\theta = 0.01, \phi = \pi/2$, then we would effectively define a new z axis in the direction $(0.01, \pi/2)$ for the next scattering.

Given this, demonstrate that this setup *can* be treated as a random walk in angle. In particular:

(a) (2 points) Determine the number of steps needed for the photon to go backward relative to its original direction. That is, how many steps does it take, on average, so that the photon is traveling at an angle $\theta > \pi/2$ if it started at $\theta = 0$ and takes random steps as described above? You only need to determine the dependence of the number of steps on $\Delta\theta$, rather than the numerical factor.

(b) (2 points) Suppose that the mean free path for each step is ℓ. Given your result from part (a), show that with enough scatterings the progress of the photon can be described as a roughly isotropic random walk with a mean free path $L \gg \ell$: effectively, the photon scatters until it randomizes its direction, with a net displacement of L from its original position, then it does so again and again. As part of your demonstration, write L as a function of ℓ and $\Delta\theta$ (again, the numerical factor is not needed).

4. (4 points total) A strange astronomical source has a photon number flux, measured at its surface, of $dN/(dE\,dA\,dt) = Ce^{-\beta E}$, where E and dE are both in units of keV, β is a pure number greater than 0, and C has units of photons cm^{-2} s^{-1}. The number flux has this form from $E = 0$ keV to $E = \infty$ keV. Thus, at the surface of the source, the number flux of photons you would measure between E keV and $E + dE$ keV is $Ce^{-\beta E}$ photons cm^{-2} s^{-1}. Derive the effective temperature $T_{\rm eff}$ for this source as measured at its surface. Express $T_{\rm eff}$ in Kelvin.