ASTR 320
Problem Set 4
Due Thursday, April 17

1. In a rotating disk, the Jeans criterion for formation of stars or planets is modified. Assume you have a gaseous disk of pure molecular hydrogen in Keplerian rotation around a star of mass M. Thus, at a given radius r, the gas is orbiting in a circle with an angular velocity $\Omega(r) = \sqrt{GM/r^3}$. Assume that the disk is vertically thin, so that at a radius r the vertical thickness is $h \ll r$. The disk has a uniform surface density of Σ g cm^{-2}; therefore, an area A of the disk has a mass ΣA. The disk also has a uniform temperature T.

(a) We derived the Jeans mass in class by requiring that the total energy of a sphere of gas be negative. In the present case, consider a circular section of radius $b \ll r$ cut out of the disk, and centered on an annulus of the disk that is a distance r from the star (you may assume that the gravitational potential of this section is $\sim -Gm^2/b$, where m is the mass of the section). What is the equivalent "Jeans mass" criterion in this case? Remember to include the effects of velocity shear as well as of temperature. Justify your criterion physically.

(b) You should find that the disk is stable for very small b and very large b; explain why, physically. You should also find that below a certain critical surface density Σ_0, there is no instability at all; calculate Σ_0 as a function of T, M, r, and the molecular mass m_{H_2}, or alternately as a function of the sound speed $c_s = \sqrt{kT/m_{H_2}}$ and disk angular velocity $\Omega(r)$. Answers to within a factor of 10 are fine.

2. Thinking more about gaseous disks, consider the following. Let a disk of gas orbit around a star of mass M. Let the mass of the gas itself be negligible, so that the only gravitational forces are those from the star. Suppose, as in the previous problem, that the gas is in Keplerian rotation at every radius, and let the disk be vertically thin.

To within a factor of three, derive the approximate thickness of the disk at a radius r, assuming that the gas is an ideal gas of temperature T made of pure atomic hydrogen. By "thickness" we mean the distance from the midplane of the disk (where the density is highest) to where the density is approximately half its maximum. **Hint:** use arguments based on statistical equilibrium to estimate how far gas could move out of the plane at temperature T.

3. In this problem we will consider another type of equilibrium, that of thermal balance. Suppose that a star is spherically symmetric. Inside the star, various processes generate energy. Consider a spherical shell inside the star, that goes from radius r to radius $r + dr$.

Let the density of gas in the shell be ρ, and the energy generation rate per unit mass at radius r be $\epsilon(r)$ (therefore, ϵ has units of erg s^{-1} g^{-1}). At a radius r, let the *net* outward flux of energy be $F(r)$ (with units of erg cm^{-2} s^{-1}). The total net outward luminosity at radius r is therefore $L(r) = 4\pi r^2 F(r)$, with units of erg s^{-1}.

Using physical arguments based on equilibrium, derive the relation that must hold for thermal balance, i.e., to prevent heat from building up or draining out of some radius.

4. Dr. I. M. N. Sane has turned his hand towards inventions. His newest idea is the "Sane pipe". This pipe has water flowing through it, and the cross sectional area of the pipe is large, then small, then large again. One is supposed to sleep with one's head on the small area portion. Dr. Sane believes that a "squeezing" effect on the water will heat up the water when it goes through the small area region, producing a nice comfy heated pillow.

You've been asked by the patent office to evaluate this invention. Water acts like an ideal fluid, and its density is very close to constant throughout the pipe. Use your intuition and the equations of hydrodynamics to determine, qualitatively, what if anything happens to the temperature of the water when it moves through the narrow portion of the pipe. **Hint 1:** the internal energy per mass w of the water is proportional to its temperature. **Hint 2:** the equation of continuity implies that if you imagine an area perpendicular to the axis of the pipe, the *total* mass per time flowing through the area is the same everywhere in the pipe.