Special topic: binary supermassive black holes

We are now in a position to address an interesting current topic in astrophysics related to binary supermassive black holes. The basic question is: since most or all galaxies have massive black holes in their centers, and galaxies often collide, with the black holes merge with each other? If so, this will provide a strong source of gravitational waves. I'm drawing on a lot of sources here, but I've been particularly enlightened by discussions with Steinn Sigurdsson (Penn State) and David Merritt (Rutgers).

Stars, with typical radii of $\sim 10^{11}$ cm, are much smaller than their typical separation ($\sim 10^{18-19}$ cm in the disks of galaxies). Therefore, in the bulk of galaxies there are rarely star-star collisions. Galaxies, however, have a non-negligible size compared to their separations: a galaxy might be 10^{23} cm in radius, and 10^{23-24} cm from its nearest neighbor galaxy. In fact, galaxies tend to cluster, so that if you have one galaxy then another one is most probably nearby. Galaxies therefore often collide, but it's a "collisionless collision" meaning that the interactions are mostly gravitational (with the minor exception of the interstellar mediums interacting, which we'll ignore because that's a small fraction of the total mass).

We know in addition that most or possibly all galaxies have supermassive black holes in their centers. These black holes have a typical mass that is about 0.2% of the mass of the central bulge of the galaxy. Currently detected masses range from around $10^6\,M_\odot$ (our Milky Way has a $2.6 \times 10^6\,M_\odot$ black hole) to several billion solar masses (the galaxy M87, in the center of the Virgo Cluster of galaxies, has a black hole of mass $3 \times 10^9\,M_\odot$). If two such black holes were to collide or merge, they would produce abundant gravitational waves, easily detectable by planned space-based instruments. But does this happen?

The first thing is that, clearly, a typical galaxy-galaxy collision will not be so precisely head-on that the central black holes hit each other directly. Angular momentum guarantees that. The collisions will instead be oblique, so on the first pass the black holes will miss each other by a lot, maybe several kiloparsecs. **Ask class:** what comes next? The initial collision is obviously strongly time-dependent. Therefore, there is a phase in which the system virializes over a few orbits. This will typically take a few hundred million years. Incidentally, collisions of the interstellar mediums with each other can produce a huge rate of star formation in this period. You see that in the Antennae, a well-known pair of colliding galaxies with lots of young stars.

Now suppose that phase is over and the system has relaxed into some kind of equilibrium. The two black holes are still far away from each other, say a couple of kiloparsecs. **Ask class:** what happens now? We found before that more massive objects tend to sink to the center of a mass distribution. This will cause the massive black holes to get closer to each other and to the center. To understand more about this, however, we need to consider the process of *dynamical friction*.

When we thought previously about the sinking of massive objects, we considered individual interactions. Now let's say that the object in question is *much* more massive than other stars, as will be the case for our black holes. Then we can think of the motion of the massive object as creating a wake as it moves past the background stars. That is, the gravity of the object will focus the background stars and give them extra energy. This energy must be taken out of the kinetic energy of the massive object. This therefore leads to a drag, or a frictional effect, even though this is pure gravity and there isn't actually any net dissipation in the system.

To be more quantitative, suppose that an object of mass M is moving with speed v in the x direction through a zero-velocity background of stars of mass $m \ll M$. Therefore, relative to the massive object, all the stars are moving with speed $-v$. Therefore, from previously, all of them get a y-velocity

$$v_y = -2GM/(vb)\ , \tag{1}$$

where b is the impact parameter. From energy conservation, the change in the speed of M is

$$Mv\Delta v = -mv_y^2/2 \Rightarrow \Delta v = -2mG^2M/(v^3b^2)\ . \tag{2}$$

As before, the number of encounters per time with stars that have impact parameters between b and $b+db$ is $vn2\pi\, b\, db$, where n is the number density of background stars, so

$$\begin{aligned} dv/dt &= \int \Delta v \cdot vn2\pi\, b\, db \\ &= -(2mG^2M/v^3)2\pi n \int b\, db\, /b^2 \\ &= -(4\pi nmG^2M/v^3)\ln(b_{\rm max}/b_{\rm min}) \end{aligned} \tag{3}$$

where as before $b_{\rm max} \sim R$, the size of the system, and $b_{\rm min} \sim GM/v^2$, for the small-angle approximation to apply. As before, we have a logarithm, so the precise limits of the impact parameter don't matter too much. This type of logarithm appears in many problems in which the force law is like $1/r^2$, such as gravity or the Coulomb force. One's ignorance about the details can be swept under this logarithm, which is often called a Coulomb logarithm. In fact, one commonly defines $\Lambda \equiv b_{\rm max}/b_{\rm min}$, so that $\ln\Lambda \sim 10$ is the Coulomb logarithm.

Note that only the product nm enters, so that it is actually the background density $\rho = nm$ that determines the rate of change in velocity (there is also a weak dependence that comes into the logarithm). The change in speed can be related to the change in kinetic energy of the massive object:

$$dE_K/dt = \frac{d}{dt}\left(\frac{1}{2}Mv^2\right) = vM(dv/dt) = -4\pi\rho(GM)^2\ln\Lambda/|v|\ . \tag{4}$$

Note that this is a genuine drag, which always operates opposite to the direction of motion. As we discussed before, orbits in a gravitational potential have negative energy, so the loss of kinetic energy causes the massive particles to sink to the center. The actual time necessary to get to the center depends on the velocity dispersion of the surrounding stars as well as M and ρ. For typical values, it's about

$$t_{\rm sink} \sim 10^{11}\,{\rm yr}(M/10^7\,M_\odot)^{-1}(r/100\ {\rm pc})^2\ . \tag{5}$$

Here we assume that the particle is at a distance r from the center. Note that this is *much* shorter than the relaxation time, because of the high mass assumed for the object.

Before resuming our analysis of binary supermassive black holes, a couple of comments about other applications. In a cluster of galaxies, dynamical friction of large galaxies against small galaxies and dark matter can cause those galaxies to sink to the center and merge. This is thought to contribute to the formation of CD galaxies, which are massive ellipticals in the center of some galaxy clusters. In globular clusters, the sinking of massive objects can cause a "gravothermal catastrophe", in which a central minicluster of high-mass stars or stellar remnants undergoes a collapse that can lead to high densities and violent interactions.

Returning to black holes, equation (5) above may seem to suggest that only the more massive black holes could merge in less than a Hubble time of $\sim 10^{10}$ yr. For example, if r is several kiloparsecs, as would be expected for a typical encounter, then two $10^6\,M_\odot$ objects might take 10^{12} yr to sink to the center. However, it is thought that in reality even such low-mass black holes will come together fairly quickly. Remembering the environments that the black holes originally inhabited, **Ask class:** can they think of what might speed up the sinking? Since a supermassive black hole is in a bulge that has ~ 500 times the mass of the black hole, at least initially the collection of matter acts together, so M is effectively 500 times larger. This causes initially rapid sinking. **Ask class:** will this continue indefinitely? No, because when the two distributions star to overlap, tidal effects will strip away the stars from around the black holes. However, by that point it is likely that the holes themselves are close enough to continue sinking towards each other in less than a few billion years.

It seems, therefore, that we've answered our question: binary supermassive black holes will happily drift towards the center, where they will eventually merge with each other. **Ask class:** is there anything that might eventually make the process of dynamical friction less efficient? Yes! At some point, when the holes are close enough to each other, they will run out of stars with which to interact. You can see the effect by considering conservation of energy. Suppose that two black holes of mass M are a distance R from each other. In that same region is a collection of stars of total mass Nm. Imagine that the black holes eventually eject *all* the stars, with small speed at infinity. The original total orbital energy of the black holes was roughly $-GM^2/(2R)$ and of the stars was of order $-G(Nm)^2/(2R)$, so if the stars end up with zero energy then the energy of the black holes in orbit is

$$\begin{aligned} -GM^2/(2R_f) &= -GM^2/(2R) - G(Nm)^2/(2R) \\ R_f &= R[M^2/(M^2+N^2m^2)] \; . \end{aligned} \tag{6}$$

Therefore, in order to make a significant change in the orbital separation of the black holes, the holes must interact with an amount of mass roughly equal to their own mass. The central densities of galaxies can be of order $10^6\,M_\odot\ \mathrm{pc}^{-3}$, so this suggests that two $10^6\,M_\odot$ black holes can only get within about 1 pc of each other before they start to run out of stars to throw to infinity.

Whoops! Does this mean that we expect many galaxies to harbor supermassive black holes orbiting around each other at a distance of a parsec? **Ask class:** what other effects might come in? There are several possibilities that have been discussed.

First, consider gravitational radiation. Two massive things orbiting around each other produces waves in spacetime that carry away energy. This loss of energy will cause a shrinkage of the orbit, and eventual coalescence. The problem is that the rate of inspiral depends very strongly on the semimajor axis (as a^4), and for typical supermassive black holes the separation needs to be < 0.01 pc before gravitational radiation can bring them the rest of the way in.

Second, think about the way in which the stars interact. In particular, imagine a single star of typical mass that comes near the binary black hole. It will feel a strongly nonaxisymmetric, time-dependent force, and will therefore be batted around before finally being ejected. However, when it is ejected it has an impact parameter that is comparable to the semimajor axis of the binary. Thus, unless it is thrown out with such force that it escapes the galaxy entirely, it will return with roughly the same impact parameter it had before (because its orbit will hardly be altered by the other stars). Each star therefore has more of an effect than you might have thought, and becomes negligible only when the binary has shrunk to a factor of a few less than its original semimajor axis, because then the star will completely miss the binary on its next orbit. This effect, while important, is not enough by itself.

Third, what about motion of the binary itself? All the interactions with stars will impart recoil to the binary, so it wanders around in the core region. In principle, if it wanders far enough, it can get fresh stars with which to interact, and all is well. In practice, however, supermassive black holes don't wander by more than 0.01 pc or so, which isn't enough.

Fourth, what about other stars that might come in to interact with the binary? Even if the binary acts like an eggbeater to kick out all of the stars originally within 0.1–1 pc of the core, there are other stars farther out that have orbits that bring them into the core. The problem here is that once those stars are exhausted, then in a spherically symmetric distribution it will effectively be forever until those orbits are replaced by other stars, meaning that no more stars come in to interact. **Ask class:** why would it take so long in a spherically symmetric potential? It's because, once the near-radial orbits are exhausted, it takes something like a relaxation time to repopulate them. That's much longer than a Hubble time. More massive objects (such as O or B stars or stellar-mass black holes) can sink more quickly, but probably not quickly enough.

Okay, so is there any other way? We assumed spherical symmetry in the distribution above; with that hint, **Ask class:** can they think of another possibility? My favorite idea among those I've heard is that if the central region of a galaxy is *not* spherically symmetric, but instead triaxial, then the orbits of the stars are box orbits. As we mentioned last time, such orbits do not individually conserve angular momentum. They can therefore pass arbitrarily close to the center. This means that it's only a few orbits until the center has lots of stars to interact with the binary, instead of $0.1N/\ln N$ orbits. I like this because it seems reasonable that after a galaxy collision one wouldn't have spherical symmetry. We'll see what future calculations have to say, but I wouldn't be surprised if this is the answer. One would then find that supermassive black holes commonly merge in the universe, which would be exciting as a source of gravitational waves and as a way to learn about strong gravity and associated extreme physics.