Summary of Lecture 8

Key points include:

1. Remember that acceleration is the rate of change in velocity (which itself is speed *and* direction of motion) at a given instant.

2. Obviously things fall differently, e.g., a sheet of paper versus a book. But Galileo showed through a set of clever experiments and *thought experiments* (by which you think about how an idealized experiment would go) that if other effects (such as air resistance) were somehow eliminated, then gravity on Earth would accelerate all things in the same way, at 10 m s^{-2} near the Earth's surface.

3. Newton generalized this to the universal law of gravitation: (a) every mass exerts gravitational force on every other mass, and the force is attractive (the force on mass A due to mass B points toward mass B), (b) that force is *directly* proportional to the product of their masses, and (c) that force is *inversely* proportional to the square of the distance between the masses. In symbols, $F_g = GM_1M_2/d^2$, where G is Newton's universal gravitational constant, M_1 is the mass of object 1, M_2 is the mass of object 2, and d is the distance between mass 1 and mass 2.

4. This simple force explains Kepler's first law (planets travel in ellipses with the center of force [the Sun] at one focus of the ellipse) and Kepler's third law (the orbital period squared is proportional to the cube of the semimajor axis of the ellipse).

5. It also *generalizes* Kepler's laws. Allowed orbital shapes are not just ellipses (of which circles are a special case), but can also be parabolas and hyperbolas. For parabolas and hyperbolas, the orbit is *unbound*, which means that the two objects in question pass by once and then never again (in contrast to ellipses or circles, where they orbit over and over again).

6. Another generalization is that Kepler's laws apply to *any* two objects in orbit around each other; it could be the Earth around the Sun, the Moon around the Earth, one of Jupiter's moons around Jupiter, an exoplanet around another star, two stars around each other, and so on. The general orbital period P for two objects with masses M_1 and M_2 and a semimajor axis a is

$$P^2 = \frac{4\pi^2}{G}\frac{a^3}{(M_1+M_2)} \ . \tag{1}$$

In the Solar System the mass of the Sun (call it M_1; the normal symbol is $M_\odot$, where $\odot$ represents the Sun) is so much larger than the mass of any planet (call it M_p) that $M_1 + M_2 = M_\odot + M_p \approx M_\odot$.

7. Because of conservation of energy and angular momentum, orbits cannot change spontaneously. That is, for an isolated two-body system (e.g., the Sun and Earth, or a satellite orbiting the Earth, or two stars around each other), the semimajor axis a and the eccentricity of the orbit are fixed. But if there is an external force (e.g., atmospheric drag, or rocket thrust, or gravity from a third object), the orbit *can* change.

8. Circular speed and escape speed. Suppose that you have an object of mass m orbiting around an object of mass $M \gg m$, and the separation between m and M is r. Then the speed of a circular orbit is $v_{\rm circ} = \sqrt{GM/r}$ and the speed at which m would escape from M's gravity (assuming that m is moving in vacuum) is $v_{\rm esc} = \sqrt{2GM/r}$. Note that neither formula depends on m because of our assumption that $M \gg m$. Earth's escape speed at its surface is ≈ 11 km s^{-1}; the Sun's is ≈ 617 km s^{-1}; and the Moon's is ≈ 2.4 km s^{-1}.

9. Gravity smooths out irregularities. For things that rotate slowly compared with their circular orbit speed, gravity drives objects toward being spherical. The stronger the gravity, the more spherical slowly-rotating things become. Thus asteroids (weak gravity) can be very lumpy, but slowly-rotating planets are very close to spherical. For an object that rotates at a significant fraction of its circular orbit speed, the shape tends toward a flattened sphere, which is why Saturn is visibly different from a sphere.

10. Gravity is responsible for tides. For example, the near part of the Earth is pulled more by the Moon than is the middle part of the Earth, so the near part of the Earth is pulled away from the middle part of the Earth and we get a *tidal bulge* toward the Moon. Less obvious but still true for the same reason is that the middle part of the Earth is pulled more by the Moon than the far part of the Earth, so the middle part of the Earth is pulled away from the far part of the Earth and we therefore *also* get a tidal bulge *away* from the Moon. This is why there are two tidal bulges per day.

11. The sloshing of the ocean (and the rest of the Earth) due to tides exerts *tidal friction* on the Earth, which slows down the Earth's rotation over time because the Moon takes longer to orbit the Earth than the Earth does to rotate. Thus, with time, the day has become longer! Over that same time, the Moon has synchronized its rotation with its orbit; the Moon is smaller than the Earth, so tidal friction affected it more profoundly. This is why we always see the same face of the Moon.