Dimensional Analysis

Step 1: Define the problem. First identify the result that you are looking for (call this A for answer). Then identify a set of variables v_i that fully constrain the problem. Specifically, be sure that if you know the values of all of these variables, then the answer that you are looking for is fully determined. Next, make sure that your variables are all independent of each other.

Step 2: From your list of variables v_i, form a quantity with the same units as A, the result that you seek. Call this Q.

Step 3: Form as many dimensionless quantities from your list of variables as you can. For instance, if your variables include a height and a length, then their ratio is a dimensionless number. If your list includes a height, width, and length, then you have two independent dimensionless quantities. Call these quantities ϕ_i.

Step 4: Multiply the quantity from Step 2 times the most general undetermined function f of your set of ϕ_i: $A = Qf(\phi_i)$. In the case where no dimensionless quantities can be formed from your variables (Step 3), then f is simply an undetermined constant. Finally, note that although dimensional analysis is a useful technique, it always gives less information that actually solving the problem.

EXAMPLE:

What is the horizontal distance D covered by a baseball thrown at a speed v at an angle θ from the horizontal?

Step 1: We are looking for a distance, D. The answer depends on v and θ, the initial conditions of the baseball on its release. The answer also depends on the strength of gravity. Since the baseball is thrown, we can assume that it doesn't go very high or far, and we can ignore things like the variation of gravity with height and the curvature of the Earth. So the answer also depends on g, the assumed constant acceleration due to gravity. The complete set of independent variables is v, θ, g, and we are looking for $D(v, \theta, g)$.

Step 2: Distance is measured in m, speed in m/s, θ in *radians* which are dimensionless, and acceleration in m/s^2. To make meters from the latter three quantities, simply divide v^2 by g. So $Q = v^2/g$.

Step 3: The only dimensionless quantity that can be formed from the variables v, θ, g is θ itself.

Step 4: The full solution to this problem must take the form $D = v^2/gf(\theta)$. The dimensional analysis is done, but does the result make sense? Check limits. If v is increased, the baseball goes farther, and if g is increased it goes less far. This seems reasonable. Convince yourself that $f(\theta) \rightarrow 0$ if either $\theta \rightarrow 0$ or $\theta \rightarrow \pi/2$.