ASTR450 Homework # 9 – Perturbations
Due Tuesday, April 22

1. **Impulse Approximation to a Drag Force.** Consider a satellite on an inclined elliptical orbit acted on by a drag force of the form $\mathbf{F} = -k\mathbf{v}_{rel}$, where $\mathbf{v}_{rel}$ is the velocity of the satellite relative to the atmosphere, and k is a positive constant. Recall that Earth's atmosphere decays exponentially with height (scale height $\approx$ 10km).
a) Approximate the range of eccentricities for which the impulse approximation will be reasonable.
b) Consider a rotating Earth. Start by making a qualitative estimate of the error made in neglecting rotation. How does the Earth's rotation affect an equatorial orbit ($i = \Omega = 0$)? Describe how the orbital elements a, e, i, Ω, ϖ vary in time.
c) Now imagine orbits with $i \neq 0, \Omega \neq 0$. Using the perturbation equations and other physical arguments, describe qualitatively how these orbits will evolve (i.e. how a, e, i, Ω, ϖ vary in time).

2. **Radial Perturbation Forces.** Consider a radial perturbation force of the form $\mathbf{F} = R\hat{r}$.
a) Apply the perturbation equations to this force and obtain simplified expressions for da/dt, de/dt, di/dt, $d\Omega/dt$, and ϖ/dt.
b) A radial perturbation to gravity, which is itself a radial force, is an example of a central force. So angular momentum must be conserved. Show that your equations conserve the angular momentum vector and describe the constraints that this imposes on these orbits.
c) Now let $R = Ar^n$ where A is a constant. Take the time average of your expressions over a single unperturbed Keplerian orbit (this step assumes that the perturbation is small). Show that $< r^n \sin\nu >= 0$ and argue, on physical ground, that $< \cos\nu >$ is negative (or zero) and that $< r^{-2}\cos\nu >= 0$. It can be shown that $< r^n \cos\nu >$ is negative for $n > -2$ and positive for $n < -2$. Use this fact to determine how the sign of your time-averaged $d\varpi/dt$ depends on A and n. Use the Central Force Integrator to check your results numerically.
d) Finally, consider the General Relativistic (GR) Perturbation $R = Ar^{-4}$ where A is a small negative constant. The integral $< r^{-4}\cos\nu >= a^{-4}e(1-e^2)^{-5/2}$. Solve, analytically, for the value of A that will give 30 degrees of precession per orbit for $e = 0.5$ (The true effects of GR on Mercury's orbit are almost exactly a million times weaker). Convert your prediction into the proper initial conditions for the Central Force Integrator, and test it! Start your orbit at pericenter.

3. **The Three Body Problem.** Use the Three-Body Integrator to investigate tadpole (paths which surround one of L_4 and L_5), horseshoe (paths which orbit both L_4 and L_5), and more complicated orbits! First, run through the Three-Body form and the graphics driver using the default values to get oriented. What do you think this tadpole orbit will look like in inertial coordinates? Find out by toggling to inertial coordinates on the graphics driver! Now you are ready to start. Take the default settings on the Three-Body form and set the velocities to zero. Your missions, should you choose to accept them, are below; each of you have a different set of initial conditions to investigate. Classify your orbits as either i) tadpole, ii) horseshoe, iii) entirely inside Jupiter's orbit, iv) entirely outside Jupiter's orbit, or v) a path which switches between several of these. Explore!

(over)

Chris: set y=0, and vary x from 0 to 3 relative to L_4
Ray: set y=0, and vary x from 0 to -3 relative to L_4
Elbert: set x=0, and vary y from 0 to 3 relative to L_4
Alaa: set x=0, and vary y from 0 to -3 relative to L_4

Mike: set y=0, and vary x from 0 to 3 relative to L_5
Jason: set y=0, and vary x from 0 to -3 relative to L_5
Linda: set x=0, and vary y from 0 to 3 relative to L_5
Lori: set x=0, and vary y from 0 to -3 relative to L_5

Plot – and think about – your orbits in both the rotating and inertial frames. Try to identify and understand the correspondence between structures seen in one frame and their representations in the other frame. Using color to represent time evolution can help you figure out what is going on! Here are some questions to think about which we will discuss in class. As you increase the distance from the Lagrange Point, does the region of horseshoe orbits change to a region of more complicated orbits in a smooth or chaotic way? Can you explain why? How about transitions between tadpole and horseshoe orbits, and between other types of orbits? What is the relationship between your orbits and the Zero Velocity Curves?