# ASTR450 Homework # 11 – Resonances and Final Review
Due Tuesday, May 6

1. Find the places along an elliptical orbit where the angle between the radius vector and the velocity vector is minimum and maximum.

2. Apollo 13 is on a circular orbit 100km above the Earth's surface. It fires its engines and produces a tangential velocity which will take it to the Moon (assume that the spacecraft's orbit is always in the Moon's orbital plane).
a) In a two-body approximation (Earth, spacecraft) how much extra velocity must be added to put the spacecraft onto an orbit whose apocenter is at the Moon? How much extra velocity does it need to escape from the Earth altogether?
b) What is the minimum extra velocity that might enable the spacecraft to get to the Moon in the three-body approximation (Earth, Moon, spacecraft)? Hint: Use Jacobi's integral to see when a transfer orbit first becomes possible.

3. Estimate the period of oscillations in Neptune's eccentricity induced by Jupiter.

4. Pluto and Neptune are linked by a 3:2 eccentricity-type resonance. The resonant term in the disturbing function felt by Pluto (due to Neptune) is $\mathcal{R}_N = M_N \beta e_P \cos(3\lambda_P - 2\lambda_N - \varpi_P)$ while the term felt by Neptune (due to Pluto) is $\mathcal{R}_P = M_P \beta e_P \cos(3\lambda_P - 2\lambda_N - \varpi_P)$. Here "P" subscripts refer to Pluto, "N" subscripts refer to Neptune, $M, e, \lambda$, and $\varpi$ are mass, eccentricity, mean longitude, and longitude of pericenter, and $\beta$ is a constant.
a) List all other first-order 2:1 resonant terms for Neptune's perturbations on Pluto. List all other second-order resonant terms.
b) Find the time rates of change of the orbital elements $(a, e, i, \Omega, \varpi)$ of Pluto and Neptune due to disturbing functions $\mathcal{R}_N$ and $\mathcal{R}_P$. Remember that $d\mathcal{R}/d\epsilon = d\mathcal{R}/d\lambda$ and $p = a(1-e^2)$. Give your answer to lowest order in eccentricity and inclination, and keep only the most perturbed element in each of these sets: $(de_N/dt, di_N/dt, de_P/dt, di_P/dt)$ and $(d\Omega_N/dt, d\varpi_N/dt, d\Omega_P/dt, d\varpi_P/dt)$. You can set the other members of each set equal to zero.
c) Show that the sum of the orbital energy of Pluto plus that of Neptune is conserved to lowest order in $e, i$.