ASTR450 EQUATION SHEET

Basic Physics:

$$\mathbf{L} = \mathbf{r} \text{ x } m\mathbf{v} \quad ; \quad \frac{d\mathbf{r}}{dt} = \dot{r}\hat{\mathbf{r}} + r\dot{\theta}\hat{\theta} \quad ; \quad \frac{d^2\mathbf{r}}{dt^2} = \left(\frac{d^2r}{dt^2} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\frac{d^2\theta}{dt^2} + 2\dot{r}\dot{\theta}\right)\hat{\theta}$$

$$\mathbf{F} = -\nabla V \quad ; \quad \frac{dE}{dt} = \int \mathbf{F} \cdot \mathbf{dv} \quad ; \quad \frac{d\mathbf{L}}{dt} = \tau = \mathbf{r} \text{ x } \mathbf{F}$$

Central Force Motion:

$\mathbf{F}$ = -f($\mathbf{r}$,$\mathbf{v}$,t)$\hat{\mathbf{r}}$ $\rightarrow$ conserves $\mathbf{h}$; $\mathbf{F}$ = -f($\mathbf{r}$)$\hat{\mathbf{r}}$ $\rightarrow$ conserves $\mathbf{h}$ and C

$$h = r^2\dot{\theta} \quad ; \quad \frac{v^2}{2} + V(\mathbf{r}) = C$$

Stability of circular orbits: $3f(a) + af'(a) > 0$ or $dh(a)/da > 0$ or $dC(a)/da > 0$

Conics:

$$r = \frac{a(1-e^2)}{1 + e\cos\nu} = a(1 - e\cos E) \quad ; \quad v = \sqrt{GM\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$h = \sqrt{GMa(1-e^2)} \quad ; \quad C = -\frac{GM}{2a} \quad ; \quad q = a(1-e)$$

Anomalies:

$$M = E - e\sin E \quad ; \quad \tan\frac{\nu}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

Jacobi's Integral:

$$C_J = \frac{G(M_1+M_2)}{R^3}\left(x^2+y^2\right) + \frac{2GM_1}{r_1} + \frac{2GM_2}{r_2} - v^2$$

Tisserand's Constant:

$$C_J \approx \frac{G(M_1+M_2)}{a} + 2\frac{G(M_1+M_2)}{R}\sqrt{\frac{a}{R}(1-e^2)}\cos i$$

Meaning of the Orbital Elements

Angle	Measured From	Measured To
Longitude	Reference Direction	-
Argument	Ascending Node	-
Anomaly	Pericenter	-
Node	-	Ascending Node
Pericenter	-	Pericenter
True	-	Planet Location
Latitude	-	Planet Location
Mean	-	Planet Location*

* If the planet moved at a constant angular rate (its mean rate).

Symmetries:

(Orbits and Equations are Unaffected by the Following Transformations)

1. Change of Reference Direction:
Longitude Angles $\rightarrow$ Longitude Angles + δL

2. Interchange of Pericenter and Apocenter:
e $\rightarrow$ -e
Pericenter Angles $\rightarrow$ Pericenter Angles + π
Anomalies $\rightarrow$ Anomalies + π

3. Interchange of Ascending and Descending Node:
i $\rightarrow$ -i
Node Angles $\rightarrow$ Node Angles + π
Arguments $\rightarrow$ Arguments + π

N-Body:

$$R_{ij} = G\left(\frac{1}{|\mathbf{r_j} - \mathbf{r_i}|} - \frac{\mathbf{r_i} \cdot \mathbf{r_j}}{\mathbf{r_j^3}}\right) \quad ; \quad \nabla R_{ij} = G\left(\frac{\mathbf{r_j} - \mathbf{r_i}}{|\mathbf{r_j} - \mathbf{r_i}|^{\mathbf{3}}} - \frac{\mathbf{r_j}}{\mathbf{r_j^3}}\right)$$

$$\mathbf{P} = \sum_{i=1}^{n} m_i \mathbf{v}_i \quad ; \quad \mathbf{L} = \sum_{i=1}^{n} m_i \mathbf{r}_i \text{ x } \mathbf{v}_i \quad ; \quad E = \sum_{i=1}^{n} \frac{1}{2} m_i v_i^2 - \sum_{i=2}^{n} \sum_{j=1}^{i-1} \frac{G m_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Quote of the Semester:

"Doing ASTR450 homework assignments is like climbing Mt. Everest once a week."